Exam 3b Calc 2 4/1/2011

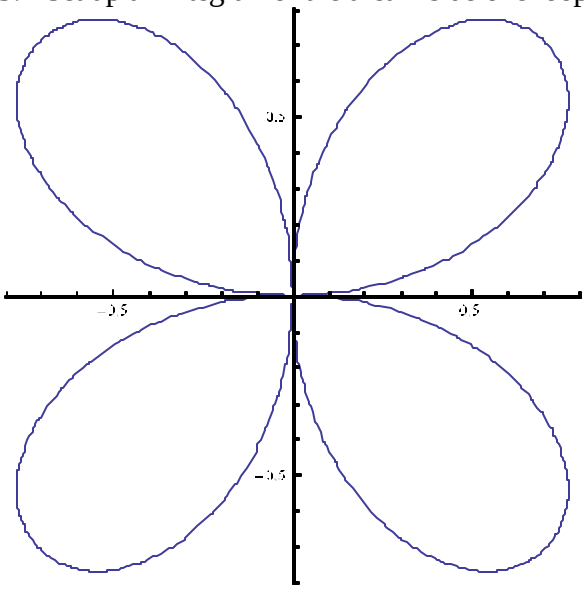
Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Write the first three terms in the sequence $\left\{\frac{n}{2n+1}\right\}$.

b) Write the first three partial sums of the series $\sum_{n=1}^{\infty}\frac{n}{2n+1}$.

2. Find the sum of the series $\frac{3}{4}-\frac{1}{2}+\frac{1}{3}-\frac{2}{9}+\frac{4}{27}-\ldots$.

3. Set up an integral for the area inside one loop of the curve $r = \sin 2\theta$.



4. Find a general solution to the differential equation $\dfrac{dy}{dx} = \dfrac{-x}{y}$.

5. Find an equation for the ellipse with vertices at (0,0), (6,0), (3,2), and (3,–2).

6. A cup of coffee starts at 160° F and cools according to the differential equation $\frac{dT}{dt} = 0.05(70 - T)$. Using Δt = 5 minutes, approximate the temperature of the coffee after 10 minutes.

7. Biff is a Calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This Euler's stuff is killing me. Look at this problem from our exam!"

> A paper cup of coffee sitting under a ceiling fan starts at 170° F and cools according to the differential equation $\frac{dT}{dt} = 0.2(70 - T)$. Using Δt = 10 minutes, approximate the temperature of the coffee after 30 minutes. Comment on the accuracy of your answer.

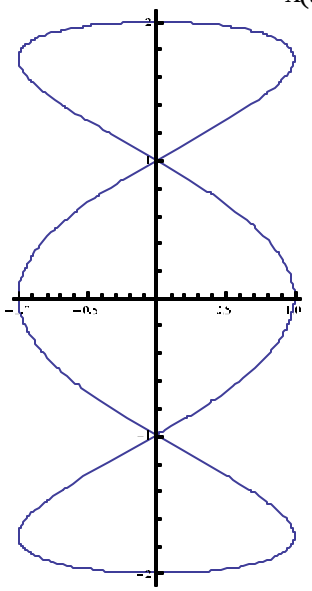
"So I worked it out, and said the answer had to be pretty good because I didn't have to round it off or anything. But when they gave the text back I got zero points for that part even though I did the number part right. Isn't math supposed to be about getting the right answer?"

Explain clearly to Biff what might have been a good comment on the accuracy of his answer.

8. Find the slope of the polar curve $r = \sin 2\theta$ (picture provided on #3).

9. Set up an integral for the area of the region enclosed by the curve with parametric equations

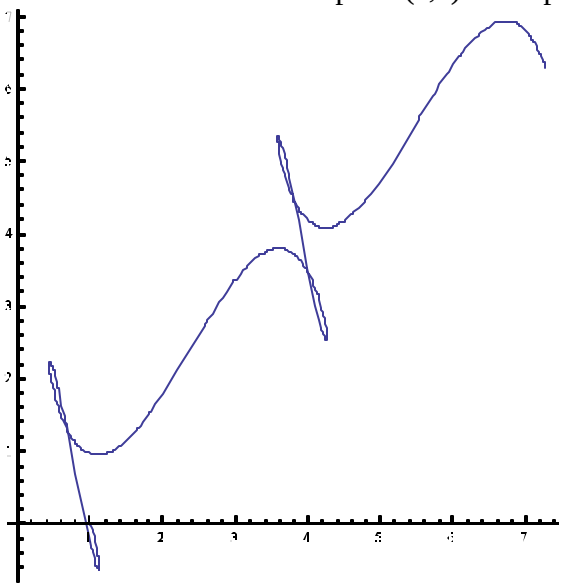
$x(t) = \cos 3t$ $\quad\quad$ $y(t) = 2\sin t$.



10. Set up an integral for the length of the portion of the curve with parametric equations

$$x(t) = t + \cos 2t \qquad y(t) = t - \sin 4t$$

which extends from the point (1,0) to the point $(2\pi + 1, 2\pi)$.



Extra Credit (5 points possible):
Evaluate the integral you set up in #10 [Hint: The identity $\sin 2x = 2\sin x \cos x$ may be useful].