Examlet 4 Foundations of Advanced Math 4/16/11

1. a) State the definition of a transitive relation.

   b) Give an example of a relation on the set $\{1, 2, 3\}$ which is symmetric but not reflexive.

2. a) Suppose that $\sim$ is an equivalence relation on the set $A = \{a, b, c, d, e\}$ and that $[a] = \{a, b\}$ and $[d] = \{d, e\}$. Write the partition $\mathcal{P}$ corresponding to $\sim$.

   b) Suppose that $\mathcal{P}$ is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Write the equivalence class of 2 under the corresponding relation.

3. Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \sim (c, d) \Leftrightarrow a - b = c - d$. Determine whether R is reflexive, symmetric, or transitive, and support your conclusions well.

4. a) The sum of the degrees of the points in a graph is always even.

b) Suppose that a graph has n vertices. What is the largest number of them that can be of degree 3?

5. a) If two relations R and S on A are reflexive, is $R \cup S$ reflexive?

b) If two relations R and S on A are transitive, is $R \cup S$ transitive?