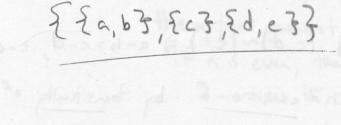
1. a) State the definition of a transitive relation.

b) Give an example of a relation on the set {1, 2, 3} which is symmetric but not reflexive.

Excellent!

2. a) Suppose that \sim is an equivalence relation on the set $A = \{a, b, c, d, e\}$ and that $[a] = \{a, b\}$ and $[d] = \{d, e\}$. Write the partition \mathcal{P} corresponding to \sim .



b) Suppose that \mathcal{P} is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Write the equivalence class of 2 under the corresponding relation.

3. Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \sim (c, d) \Leftrightarrow a - b = c - d$. Determine whether R is reflexive, symmetric, or transitive, and support your conclusions well.

REFLEXIVE

For R to be reflexive then $(a,b) \sim (a,b)$ So, a-b=a-b and this is true: R is reflexive

SYMMETEIC

For R to be symmetric then $(a,b) \wedge (c,d) \Rightarrow (c,d) \wedge (a,b)$ So, a-b=c-d which we know is true. Then we have that c-d=a-b which we also know is true because it is the same as a-b=c-d just written opposite, o' R is symmetric

TRANSITIVE

For R to be transitive, $(a,b)^{n}(c,d)$ and $(c,d)^{n}(e,f) \ni (a,b)^{n}(e,f)$ So, we know a-b=c-d and c-d=e-f. We can we do some substitution because c-d=e-f and a-b, and we get that a-b=c-f. R is transtite

Ris an equivallance relation because it is retlexive, symmetric and transitive

Excellent!

4. a) The sum of the degrees of the points in a graph is always even.

Let's induct on the number of ages in the graph. If n=1, there are 2 vertices of degree 1 (and possibly others of degree 0) for a total degree of 2, which is even.

Now sipose it's frue when you have k edges, and consider a graph with k+1 edges. Adding the k+1st edge adds one to the degree of each vertex in that edge, we adds 2 to the even sum of the degrees of vertices in our graph with k edges, preserving and even sum. I

b) Suppose that a graph has n vertices. What is the largest number of them that can be of degree 3?

For a less than 4, it's 0 because there aren't enough other vertices. For even $n \ge 4$, all n can be of degree 3: n = 4n=6

For odd n = 5, n-1 can be of degree 3. It can't be all n since that would violate parta, but we can add a vertex to the lower outside edge in the scheme above to attain n-1. I 5. a) If two relations R and S on A are reflexive, is $R \cup S$ reflexive? $\forall a \in A$, since R is reflexive we have $(a,a) \in R$ and thus $(a, \cdot) \in R \cup S$, so $R \cup S$ is reflexive. \square

b) If two relations R and S on A are transitive, is $R \cup S$ transitive?

No. For a counterexample consider $A = \{1, 2, 3\}$, with $R = \{(1, 2)\}$ and $S = \{(2, 3)\}$. We have R and S both vacuously transitive, but $(1, 2) \in R \cup S$ and $(2, 3) \in R \cup S$ even though $(1, 3) \notin R \cup S$. \square

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