Problem Set 4   Foundations   Due 2/21/2011

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Prove that for any set A, $\varnothing \subseteq A$.

2. Prove that the empty set is unique.

3. a) Let $A = \{a, b, c\}$. Find $\mathcal{P}(A)$.

   b) Let $A = \{a, b, c, d\}$. Find $\mathcal{P}(A)$.

4. Prove that if A has n elements, then $\mathcal{P}(A)$ has 2^n elements. [Hint: Induction!]

5. Show that $\left(\bigcup_{i \in I} A_i \right)' = \bigcap_{i \in I} A_i'$.

6. Show that $A \cap \bigcup_{i \in I} B_i = \bigcup_{i \in I} (A \cap B_i)$.