Problem Set 7   Foundations   Due 4/4/2011

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. For any a, b, c, $d \in \mathbb{R}$ with $a < b$ and $c < d$, the sets (a,b) and (c,d) are equipollent.

2. The sets $(0,1)$ and $(-\infty, \infty)$ are equipollent.

3. If A and B are disjoint denumerable sets, then $A \cup B$ is denumerable.

4. If A and B are denumerable sets, then $A \cap B$ is denumerable.

5. Let R be the relation on $\mathbb{Z}$ defined by $x \, R \, y$ iff $|x - y| < 5$.
   a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.

6. Determine whether the relation R from problem 5 is reflexive, symmetric, and transitive.

7. Let S be the relation on $\mathbb{Z}$ defined by $x \, S \, y$ iff $x - y$ is odd.
   a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.

8. Determine whether the relation S from problem 5 is reflexive, symmetric, and transitive.