

The instructors will select four of these problems to grade, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. For $n \geq 1$ and $1 \leq k \leq n$, $C(n, k) = \frac{n!}{k!(n-k)!}$.

2. For $n \geq 1$ and $1 \leq k \leq n$, $k \cdot C(n, k) = n C(n-1, k-1)$.

3. The Banana Theorem: Let A be a set with n elements of k different types (such that elements of the same type are regarded as indistinguishable from one another for purposes of orderings). Let n_i be the number of elements of type i for each integer i from 1 to k . Then the number of different

arrangements of the elements in A will be $\frac{n!}{\prod_{i=1}^k (n_i!)}$.

4. How many numbers between 1 and 1,000,000 have exactly three 9's among their digits?
5. If a jar contains five balls, one red, two blue, and two white, and two balls are drawn at random from the jar (replacing each after it's drawn), what is the probability that both balls are white?
6. If a jar contains five balls, one red, two blue, and two white, and two balls are drawn at random from the jar (*not* replacing each after it's drawn), what is the probability that both balls are white?
7. If a jar contains five balls, one red, two blue, and two white, and three balls are drawn at random from the jar (*not* replacing each after it's drawn), what is the probability that the balls are drawn red, blue, and white in that order?
8. "In the jungle, you must wait, until the dice read 5 or 8" – from the movie *Jumanji*
What is the probability of a 5 or 8 total when two standard dice are rolled?

