Exam 2a Calc 2 3/2/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. For $\int_0^1 \sin\left(\sqrt{x}\right) dx$, the left-hand approximation using $n = 2$ subintervals is 0.3248 (to four decimal places). Find the midpoint and trapeziodal approximations with $n = 2$ subintervals.

2. Evaluate $\int \sin^5 \theta \, d\theta$.

3. **Set up** an integral for the surface area obtained by rotating the curve $y = 1/x$ on $[1,10]$ around the x-axis.

4. Find the present value of an income stream of \$2000 per year, for a period of 10 years, if the continuous interest rate is 5%.

5. Evaluate $\displaystyle\int_8^\infty \frac{dx}{\sqrt[3]{x}}$.

6. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is sooooo hard. I understand when they ask you to, like, work out a probability or something, right? But there was this problem on our test about why this one function wasn't a probability dense function, and that's totally unfair. How am I supposed to know it isn't one? The function was, like, $p(x) = \begin{cases} 0.2 & \text{for } 0 \le x \le 6 \\ 0 & \text{for } x<0 \text{ or } x>6 \end{cases}$."

   Explain clearly to Bunny how one can tell whether a function like this is (or is not) a p.d.f..

7. Suppose the function $p(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.4e^{-0.4x} & \text{for } x \ \geq \ 0 \end{cases}$ is a probability distribution function for the probability that a shirt lasts x years before getting torn. Find the median number of years a shirt lasts.

8. Show that $\int \sqrt{a^2 + x^2}\,dx$ can be transformed into $a^2 \int \sec^3 \theta \, d\theta$ by an appropriate substitution.

9. Derive the reduction formula $\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$ (provided $n \neq 1$).

10. Evaluate $\int \frac{1}{x\left(x^2+1\right)}\,dx$.

Extra Credit (5 points possible):

Evaluate $\int_0^{\pi} \sin^m x\,dx$ in terms of *m*.