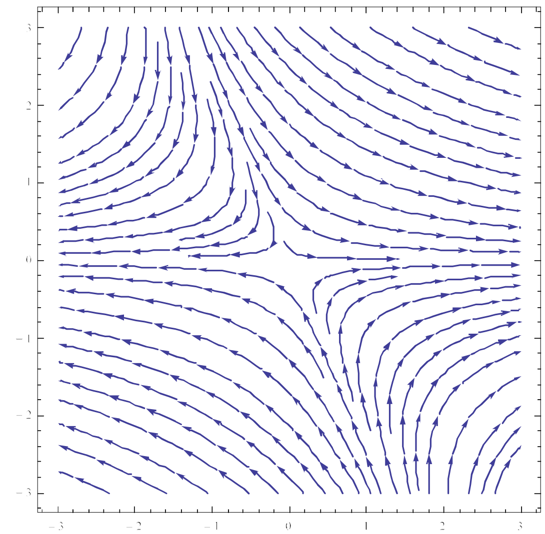
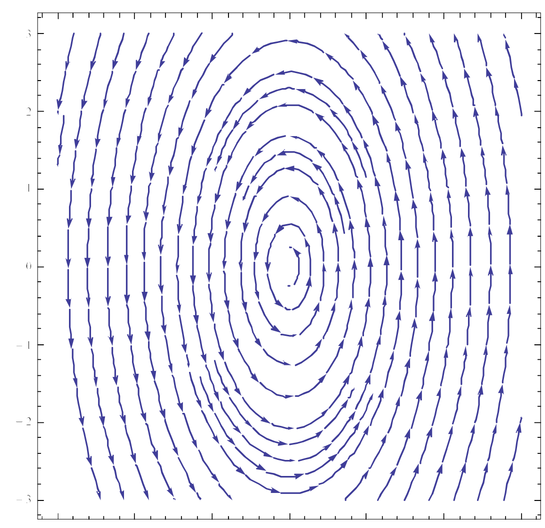
Exam 3   Differential Equations   4/13/12

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. One of the planar systems whose phase plane is shown at right has two real eigenvalues, and the other has purely imaginary eigenvalues. Identify which is which.





2. State the Great Theorem of Page 305.

3. If a planar system of differential equations has eigenvalues $\lambda_1 = 3$, $\lambda_2 = 1$ and associated eigenvectors $\mathbf{v}_1 = (1,0)$ and $\mathbf{v}_2 = (-2,1)$, write a general solution to the system.

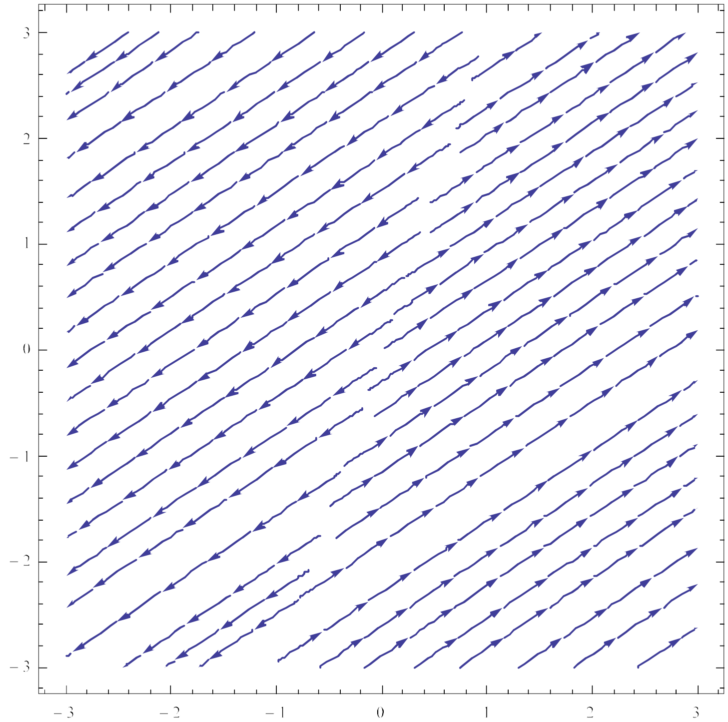
4. Find a solution to the initial-value problem $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$, $y(0) = 0$, $y'(0) = 2$.

5. Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 0 & 2 \\ -2 & -1 \end{pmatrix}\mathbf{Y}$. Is the origin a spiral source, spiral sink, or center?

6. Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}\mathbf{Y}$. Find a general solution to this system.

7. State and prove the Bandicoot Theorem.

8. Consider the family of linear systems $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & -1 \\ 2 & d \end{pmatrix}\mathbf{Y}$. For which values of d will the vector field look like the one below?



9. Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & 10 \\ -1 & 3 \end{pmatrix}\mathbf{Y}$. Find a general solution to this system.

10. Consider the family of linear systems $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & b \\ -2 & 1 \end{pmatrix}\mathbf{Y}$.

a) For which values of b will solutions oscillate?

b) For which values of b will solutions move away from the origin for all initial conditions other than (0,0)?