Examlet 4 Foundations of Advanced Math 4/13/12

1. a) State the definition of a transitive relation.

A relation $\sim$ on set S is transitive if, $\forall a, b, c \in S$, if $a \sim b \wedge b \sim c$, then $a \sim c$. $\square$ Great

b) Give an example of a relation on the set $\{1, 2, 3\}$ which is reflexive but not symmetric.

$\{(1,1), (2,2), (3,3), (1,2)\}$ Excellent!

This is reflexive since $1 \sim 1$, $2 \sim 2$, and $3 \sim 3$, but it is not symmetric since $1 \sim 2$ but $2 \not\sim 1$. $\square$

2. a) Suppose that $\sim$ is the relation on the set $A = \{a, b, c, d, e\}$ given by $\{(a, a), (a, c), (a, e), (b, b), (c, a), (c, c), (c, e), (d, d), (e, a), (e, c), (e, e)\}$. Write the partition $\mathcal{P}$ corresponding to $\sim$.

$[a] = \{a, c, e\}$
$[b] = \{b\}$
$[c] = \{a, c, e\}$
$[d] = \{d\}$
$[e] = \{a, c, e\}$

$\mathcal{P} = \{\{a, c, e\}, \{d\}, \{b\}\}$ Great!

b) Suppose that $\mathcal{P}$ is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Write the equivalence class of 2 under the corresponding relation.

$[2] = \{2, 4\}$

Yes

3. Let R be a relation on $\mathbb{Z}$ defined by $a \equiv b$ iff $a = 3b$. Determine whether ~ is reflexive, symmetric, or transitive.

Reflexive

If ~ is reflexive then it must hold true for $a \sim a$, $a = 3a$ which is not true so ~ is not reflexive.

Counter example: $a = 1$

$1 \neq 3$

Symmetric

If ~ is symmetric than the relation must hold true for $a \sim b$ and $b \sim a$

so if $a = 3b$ than $b = 3a$

Counter example:

$a = 3 \quad b = 1$

$3 = 3(1)$ but $1 \neq (3)(3)$

Wonderful!

So ~ is not symmetric

Transitive

If ~ is transitive than the relation must hold true for: $a \sim b$ and $b \sim c$ than $a \sim c$

So: $a = 3b$ and $b = 3c$

use substitution: $a = 3(3c)$

counter example: $a = 9$, $b = 3$, $c = 1$

$9 = 3 \times 3$ (true) $3 = 3(1)$ true but $9 \neq 3 \times 1$ false

So ~ is not transitive.

4. a) Express the definition of the sum of two functions in terms of ordered pairs.

If $(a, b_1) \in F_1$ and $(a, b_2) \in F_2$, then $(a, b_1 + b_2) \in F_1 + F_2$. $\square$

Good

b) Express the definition of the composition of two functions in terms of ordered pairs.

If $(a, b) \in f$ and $(b, c) \in g$, then $(a, c) \in g \circ f$. $\square$

Excellent

5. Every cubic graph has an even number of vertices.

Well, by previous proof we know the number of odd-degree vertices on a graph must be even. Since every vertice in a cubic graph has an odd degree, there must be an even number of total vertices in the graph. □

Nice.