Examlet 4B Foundations of Advanced Math 4/19/13

1. a) State the definition of a reflexive relation.

   b) Give an example of a relation on the set $\{a, b, c\}$ which is transitive but not symmetric.

2. Let $S = \{1, 2, 3, 4, 5\}$, and consider the partition $\mathcal{P} = \{\{1,2\},\{3,5\},\{4\}\}$ of S. Write the equivalence relation $\sim$ corresponding to $\mathcal{P}$.

3. a) Express the definition of the sum of two functions $f, g : \mathbb{R} \to \mathbb{R}$ formally in terms of ordered pairs.

b) Express the definition of a surjection formally in terms of ordered pairs.

4. Let S be a set and $\mathcal{P}$ a partition of S.
   a) The relation on S defined by $a \sim b$ iff $\exists P \in \mathcal{P}$ for which $a, b \in P$ is a reflexive relation.

   b) The relation on S defined by $a \sim b$ iff $\exists P \in \mathcal{P}$ for which $a, b \in P$ is a symmetric relation.

   c) The relation on S defined by $a \sim b$ iff $\exists P \in \mathcal{P}$ for which $a, b \in P$ is a transitive relation.

5. Say that two vertices v_1 and v_2 of a graph G are **adjacent** iff there exists a walk with exactly one edge between them.

   a) Is the relation of being adjacent reflexive?

   b) Is the relation of being adjacent symmetric?

   c) Is the relation of being adjacent transitive?