Examlet 2 Advanced Geometry 3/13/13

1. a) State the definition of an *exterior angle* of a triangle.

An exterior angle of a triangle is an angle That forms a linear pair with one of the triangle's interior angles.

Great

b) State the definition of $\sigma(\Delta ABC)$.

The angle sum of a triangle is the sum of The measures of the interior angles.

$\sigma(\Delta ABC) = \mu(\angle ABC) + \mu(\angle BCA) + \mu(\angle CAB)$.

Excellent

c) State the definition of a *convex* quadrilateral.

A quadrilateral is convex if each vertex is in the interior of the angle created by the other three points.

Nice

quad:

Let A, B, C, D be 4 points s.t. any three points do not lie on any one line. Suppose that any two of the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ either do not have any points in common or have only an endpoint in common. If these criteria are met then $\square ABCD$ is a quadrilateral.

2. a) State the Exterior Angle Theorem.

An exterior angle of a triangle is strictly greater than each of the triangle's remote interior angles.

Great

b) State the Converse to the Isosceles Triangle Theorem.

For a triangle $\triangle ABC$ if $\angle ABC \cong \angle ACB$ then $\overline{AB} \cong \overline{AC}$.

Good.

c) State the Scalene Inequality.

In any triangle the greater side lies opposite the greater angle and the greater angle lies opposite the greater side.

Great

d) State the Saccheri-Legendre Theorem.

The sum of the angles of a triangle is less than or equal to $180°$.

$(\sigma(\triangle ABC) \leq 180°)$

Great

e) State Hilbert's Parallel Postulate.

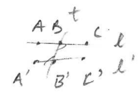
For a line l and external point P $\exists$ at most one line m $\ni$ P lies on m and $m \parallel l$.

Great

3. State five conditions that are equivalent to the Euclidean Parallel Postulate.

1. Clairaut's Axiom: $\exists$ a rectangle
2. Angle Sum Postulate: $\sigma(\triangle ABC) = 180°$ for any triangle
3. Hilbert's Parallel Postulate: on previous page
4. Wallis's Postulate: For a triangle $\triangle ABC$ and line segment $\overline{DE}$, $\exists$ a point $F \ni$ $\triangle ABC \sim \triangle DEF$
5. Transitive Property of Parallels: If $l \parallel m$ and $m \parallel n$, then either $l = n$ or $l \parallel n$ where l, m, n are lines.

Good



Alt int Angle Thm

4. Prove that if ℓ and ℓ' are two lines cut by a transversal t in such a way that a pair of alternate interior angles is congruent, then ℓ is parallel to ℓ'.

Let l and l' be two lines cut by transversal t s.t. a pair of alternate interior angles is congruent (hyp). Let points A, B, C and A', B', C' be defined as in the def of transversal, and let $\angle A'B'B \cong \angle B'BC$ (hyp).

Suppose $\exists$ a point D s.t. D lies on both l and l' (RAA hyp). We know D must lie on one side of t (Plane Sep Post) and it can't lie on t (def of transversal).

So, first suppose that D lies on the same side of t as C (RAA hyp1). Then we have $\angle A'B'B$ is an exterior angle to triangle $\triangle BB'D$ with $\angle B'BD = \angle B'BC$ being a remote int angle (def of ext and remote int angles). So, by Ext angle Thm we know $\mu(\angle A'B'B) > \mu(\angle B'BC)$ which contradicts our hypothesis, so we reject RAA hyp1 so D cannot lie on the same side of t as C.

So, the only other option is for D to lie on the same side of t as A (RAA hyp2). This gives us that $\angle B'BC$ is an ext angle (def of ext and remote int angles) to triangle $\triangle B'BD$ with $\angle DB'B = \angle A'B'B$ being a remote int angle. So by the Ext angles Thm we have $\mu(\angle B'BC) > \mu(\angle A'B'B)$ which contradicts our hypothesis, so we reject RAA hyp2 so D can't lie on the same side of t as A.

Thus, we reject RAA hyp $\nexists$ a point D s.t. D lies on both l and l' so l and l' are parallel (def of parallel).

Excellent.

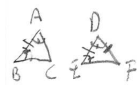
W

Notice that if one pair of alt int angles is congruent than the other pair also is congruent.

5. Provide good justifications in the blanks below for the corresponding statements:

Angle-Side-Angle Theorem: If two angles and the included side of one triangle are congruent to the corresponding parts of a second triangle, then the two triangles are congruent.

Restatement: If ΔABC and ΔDEF are two triangles such that $\angle CAB \cong \angle FDE$, $\overline{AB} \cong \overline{DE}$, and $\angle ABC \cong \angle DEF$, then $\Delta ABC \cong \Delta DEF$.



Statement:	Reason:
Let ΔABC and ΔDEF be two triangles such that $\angle CAB \cong \angle FDE$, $\overline{AB} \cong \overline{DE}$, and $\angle ABC \cong \angle DEF$	Hypothesis!
There exists a point C' on $\overrightarrow{AC}$ such that $\overline{AC'} \cong \overline{DF}$	Point Construction Postulates
Now $\Delta ABC' \cong \Delta DEF$	(SAS) Postulate.
and so $\angle ABC' \cong \angle DEF$.	Definition of Congruent triangles
Since $\angle ABC \cong \angle DEF$	by hypothesis
we can conclude $\angle ABC \cong \angle ABC'$.	Transitive Property
Hence $\overrightarrow{BC} \cong \overrightarrow{BC'}$	Protractor Postulate Unique Rays!? Yes!
But $\overrightarrow{BC}$ can only intersect $\overleftrightarrow{AC}$ in at most one point	Theorem 3.1.7
so $C = C'$ and the proof is complete.	Because it's complete.