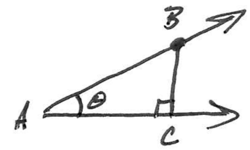
Examlet 3 Advanced Geometry 4/12/13

1. a) State the definition of sin θ for an acute angle θ.

Let θ be an acute angle, and call its vertex A. Pick a point on one of its rays and label it B, then drop a perpendicular to the other ray and call its foot C. Now define

$$\sin\theta = \frac{BC}{AB}$$



b) State the definition of a *square*.

A square is a quadrilateral that is both a rectangle and a rhombus.

c) State the definition of the *interior* of ΔABC.

The interior of ΔABC is the intersection of the interiors of the three angles $\angle ABC$, $\angle BCA$, and $\angle CAB$.

2. a) State the Fundamental Theorem on Similar Triangles.

If $\triangle ABC$ and $\triangle DEF$ are 2 $\triangle$'s s.t. $\triangle ABC \sim \triangle DEF$,

then $\frac{AB}{AC} = \frac{DE}{DF}$

Good

b) State the Law of Cosines.

For any $\triangle ABC$, $c^2 = a^2 + b^2 - 2ab\cos C$.

Good

c) State the Neutral Area Postulate.

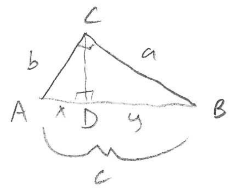
For each polygonal region R, there is an associated non-negative # $\alpha(R)$, called the area of R, s.t. the following conditions are satisfied:

① congruence: If 2 polygons have congruent polygonal regions, then the polygonal regions have equal areas.

② additivity: If the polygonal region R is the union of 2 polygonal regions, R_1 & R_2, that do not overlap, then $\alpha(R) = \alpha(R_1) + \alpha(R_2)$.

Excellent

3. Prove the Pythagorean Theorem.

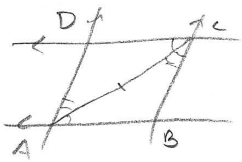


Let $\triangle ABC$ be a right triangle with right angle at $\angle C$ (hyp). Drop a perpendicular from C to $\overleftrightarrow{AB}$ and we know the foot, D, is on $\overline{AB}$ (4.8.6). By the angle sum thm we know $\mu(\angle A)+\mu(\angle B)=90^\circ$ and we know $\mu(\angle A)+\mu(\angle ACD)=90^\circ$. Thus $\mu(\angle B)=\mu(\angle ACD)$ (alg). Similarly we can find that $\mu(\angle A)=\mu(\angle BCD)$ (Angle sum thm). Thus, by definition we know $\triangle ABC \sim \triangle ACD \sim \triangle CBD$.

Let $x=AD$, $y=DB$. We know by the similar triangles thm that $\frac{x}{b}=\frac{b}{c}$ and $\frac{y}{a}=\frac{a}{c}$ so we know $b^2=xc$ and $a^2=yc$ (alg). Adding these equations, we get $a^2+b^2=c(x+y)$ but since $x+y=c$ we have $a^2+b^2=c^2$ (sub). $\square$

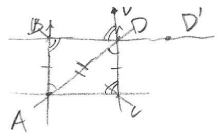
Well done

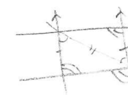
4. Show that if $\square ABCD$ is a parallelogram, then the opposite sides are congruent.



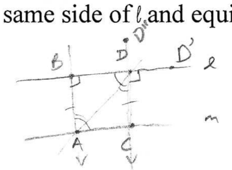
Let $\square ABCD$ be a parallelogram. We know $\overleftrightarrow{AC}$ is a transversal that cuts parallel lines $\overleftrightarrow{DC}$ and $\overleftrightarrow{AB}$ so $\angle DCA \cong \angle BAC$ (conv alt int angles). Similarly $\overleftrightarrow{AC}$ is a transversal that cuts parallel lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ so $\angle DAC \cong \angle ACB$ (conv alt int Angles). And since $\overline{AC} \cong \overline{AC}$ we have $\triangle ACD \cong \triangle CAB$ (ASA). Thus, by the definition of congruent triangles we know corresponding sides are congruent, that is $\overline{DC} \cong \overline{AB}$ and $\overline{AD} \cong \overline{BC}$. $\square$

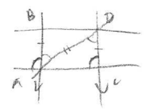
Excellent.

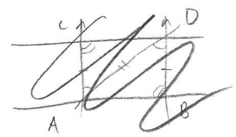




5. Prove that if ℓ and m are distinct lines and there exist two different points of m that are on the same side of ℓ and equidistant from ℓ, then $\ell \parallel m$.







Well, let A and C be the two points on m that are equidistant from ℓ. Then by the existance and uniqueness of perpendiculars we know there are lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ that are perpendicular to ℓ in such a way that B and D are the foots of the perpendiculars respectively. Thus we have that $\angle ABD \cong \angle CDD'$ (where D' is a point as shown in the figure). By Corresponding Angles Thm we know $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Since A and C are equidistant from ℓ, we know $\overline{AB} \cong \overline{CD}$. We know $\overleftrightarrow{AD}$ is a transversal cutting two parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ so conv to alt int angles tells us $\angle BAD \cong \angle CDA$, and since $\overline{AD} \cong \overline{AD}$, then SAS tells us $\triangle ADB \cong \triangle DAC$. Thus by the def of congruent triangles we know $\angle ACD \cong \angle DBA$. And since $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{BD}$ is a transversal cutting them we know by conv to the alternate int angles thm that $\angle ABD \cong \angle BDD''$ (where D'' is defined as in the figure). Thus we have $\angle ACD \cong \angle ABD \cong \angle BDD''$ so by corresponding angles thm we know $\overleftrightarrow{BD} \parallel \overleftrightarrow{AC}$, thus $\ell \parallel m$.

Good!