## Exam 3 Differential Equations 4/18/14

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Does the planar system whose phase plane is shown below have two real eigenvalues, or two complex eigenvalues? What are the signs on the real portions of the eigenvalues?

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4. If a planar system of differential equations has eigenvalues $\lambda_{1}=-4, \lambda_{2}=2$ and associated eigenvectors $\mathbf{v}_{1}=(1,2)$ and $\mathbf{v}_{2}=(-1,1)$, write a general solution to the system.
5. Find the solution (in scalar form) of the initial-value problem $\frac{d^{2} y}{d t^{2}}+5 \frac{d y}{d t}+6 y=0$, with $y(0)=0, y^{\prime}(0)=3$.
6. Consider the system $\frac{d \mathbf{Y}}{d t}=\left(\begin{array}{ll}3 & 4 \\ 1 & 0\end{array}\right) \mathbf{Y}$. Find a general solution to this system.
7. State and prove the Bandicoot Theorem.
8. Consider the system $\frac{d \mathbf{Y}}{d t}=\left(\begin{array}{cc}2 & 1 \\ -1 & 4\end{array}\right) \mathbf{Y}$. Find a solution to this system satisfying the initial condition $\mathbf{Y}(0)=(1,0)$.
9. Consider the system $\frac{d \mathbf{Y}}{d t}=\left(\begin{array}{cc}-1 & -2 \\ 1 & -4\end{array}\right) \mathbf{Y}$. If Jon wants to alter this system slightly in order not to have any distinguished lines in the phase plane, what values could Jon replace the -2 in the top right corner with to accomplish this?
10. Find the general solution to the linear system $\frac{d \mathbf{Y}}{d t}=\left(\begin{array}{ll}-2 & 3 \\ -2 & 2\end{array}\right) \mathbf{Y}$, and then the particular solution satisfying $\mathbf{Y}(0)=\binom{3}{2}$.
