Fake Problem Set 1 Differential Equations Due 4/14/14

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit you must submit something.

1. [B/D/H 3rd] Find the solution (in scalar form) of the initial-value problem $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=0$, with $y(0)=0$, $y'(0)=2$.

   The general solution is $y(t)=Ae^{-2t}+Be^{-3t}$, and our particular solution is $y(t)=2e^{-2t}-2e^{-3t}$.

2. [B/D/H 3rd] Find the solution (in scalar form) of the initial-value problem $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$, with $y(0)=1$, $y'(0)=1$.

   The general solution is $y(t)=Ae^{-t}+Bte^{-t}$, and our particular solution is $y(t)=e^{-t}+2te^{-t}$.

3. [B/D/H 3rd] Find the general solution to the linear system $\frac{d\mathbf{Y}}{dt}=\begin{pmatrix}1 & 3\\ 1 & -1\end{pmatrix}\mathbf{Y}$, and then the particular solution satisfying $\mathbf{Y}(0)=\begin{pmatrix}-2\\ 3\end{pmatrix}$.

   The general solution is $\mathbf{Y}(t)=Ae^{2t}\begin{pmatrix}3\\ 1\end{pmatrix}+Be^{-2t}\begin{pmatrix}1\\ -1\end{pmatrix}$, and our particular solution is

   $$\mathbf{Y}(t)=\frac{1}{4}e^{2t}\begin{pmatrix}3\\ 1\end{pmatrix}-\frac{11}{4}e^{-2t}\begin{pmatrix}1\\ -1\end{pmatrix}.$$

4. [B/D/H 3rd] Find the general solution to the linear system $\frac{d\mathbf{Y}}{dt}=\begin{pmatrix}-2 & 3\\ -2 & 2\end{pmatrix}\mathbf{Y}$, and then the particular solution satisfying $\mathbf{Y}(0)=\begin{pmatrix}-2\\ 2\end{pmatrix}$.

   The general solution is

   $$\mathbf{Y}(t)=A\begin{pmatrix}3\cos\sqrt{2}\,t\\ 2\cos\sqrt{2}\,t-\sqrt{2}\sin\sqrt{2}\,t\end{pmatrix}+B\begin{pmatrix}3\sin\sqrt{2}\,t\\ \sqrt{2}\cos\sqrt{2}\,t+2\sin\sqrt{2}\,t\end{pmatrix},$$ and the solution to the initial-value problem is $$\mathbf{Y}(t)=\begin{pmatrix}-2\cos\sqrt{2}\,t+5\sqrt{2}\sin\sqrt{2}\,t\\ 2\cos\sqrt{2}\,t+4\sqrt{2}\sin\sqrt{2}\,t\end{pmatrix}.$$