**Examlet 4 Foundations of Advanced Math 4/18/14**

1. a) State the definition of a relation from $A$ to $B$.

b) State the definition of a partition of a set $A$.

c) State the definition of a graph.

2. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $a \sim b$ iff $a - b$ is threven. Show that $\sim$ is an equivalence relation, being clear about your reasoning.

3. a) Express the definition of a surjective function formally in terms of ordered pairs.

b) Express the definition of an even function formally in terms of ordered pairs.

4. a) Let $S$ be a set and $\Pi$ a partition of $S$ defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a reflexive relation.

b) Let $S$ be a set and $\Pi$ a partition of $S$ defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a symmetric relation.

c) Let $S$ be a set and $\Pi$ a partition of $S$ defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a transitive relation.

5. a) In any graph, the number of vertices of odd degree is even.

b) If a graph $G$ is connected, then the graph $G'$ having the same vertex set and an edge set with exactly one fewer element is also connected.