Examlet 2 Advanced Geometry 3/4/15

1. a) State the definition of an *exterior angle* of a triangle.

   b) State the definition of a convex quadrilateral.

   c) State the Saccheri-Legendre Theorem

   d) State Euclid's Postulate V.

   e) State the Universal Hyperbolic Theorem

2. How do you know ASS (Angle-Side-Side) is not a valid triangle congruence condition?

3. a) Provide good justifications in the blanks below for the corresponding statements:

Proposition: Let ΔABC be a triangle. If $AB > BC$ then $\mu(\angle ACB) > \mu(\angle BAC)$.

Statement:	Reason:
Let A, B, and C be three noncollinear points. Let $AB > BC$.	
Since $AB > BC$, there exists a point D between A and B such that $\overline{BD} \cong \overline{BC}$.	
Now $\mu(\angle ACB) > \mu(\angle DCB)$,	
and $\mu(\angle DCB) \cong \mu(\angle CDB)$.	
But $\angle CDB$ is an exterior angle for ΔADC, so $\mu(\angle CDB) >\ (\angle CAB)$.	
The conclusion follows from those inequalities.	The conclusion follows from those inequalities.

b) Let ΔABC be a triangle. Show that if $\mu(\angle ACB) > \mu(\angle BAC)$ then $AB > BC$.

4. Provide good justifications in the blanks below for the corresponding statements:

Proposition: If there exists one line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0, then for every line ℓ and for every external point P there exist at least two lines that pass through P and are parallel to ℓ.

Statement:	Reason:
S'pose there exists a line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0.	
Then the Euclidean Parallel Postulate fails.	
No rectangle exists.	
Let ℓ be a line and P an external point.	
We must prove that there are at least two lines through P that are both parallel to ℓ. Drop a perpendicular to ℓ through P and call the foot of that perpendicular Q.	
Let m be the line through P that is perpendicular to $\overleftrightarrow{PQ}$.	
Choose a point R on ℓ that is different from Q and let t be the line through R that is perpendicular to ℓ.	
Drop a perpendicular from P to t and call the foot of the perpendicular S.	
Now $\square PQRS$ is a Lambert quadrilateral.	
But $\square PQRS$ is not a rectangle (reason?), so $\angle QPS$ is not a right angle and $\overleftrightarrow{PS} \neq m$.	
Nevertheless $\overleftrightarrow{PS}$ is parallel to ℓ,	
so our proof is complete.	Because our proof is complete.

5. A **rhombus** is a quadrilateral with four congruent sides, and a **square** is a quadrilateral with four congruent sides and four right angles.

   a) Do rhombi exist in neutral geometry? [Hint: Let $\overline{AB}$ and $\overline{CD}$ be segments that share a common midpoint, with $\overline{AB} \perp \overline{CD}$, and look at $\square ACBD$.]

   b) Do squares exist in neutral geometry?