Examlet 3   Advanced Geometry   4/10/15

1. a) State the Fundamental Theorem on Similar Triangles.

   b) State the definition of cos θ for an acute angle θ.

   c) Let ΔABC be a triangle. State the definition of the associated triangular region.

2. a) State the Neutral Area Postulate.

   b) State the Euclidean Area Postulate.

3. State and prove the Law of Sines.

4. a) State a theorem (of your choice) from §6.2 about common perpendiculars.

b) Provide good justifications in the blanks below for the corresponding statements:

Proposition: In hyperbolic geometry, if $\Delta ABC \sim \Delta DEF$, then $\Delta ABC \cong \Delta DEF$.

Statement:	Reason:
Let ΔABC and ΔDEF be two triangles such that $\Delta ABC \sim \Delta DEF$.	
If any one side of ΔABC is congruent to the corresponding side of ΔDEF , then $\Delta ABC \cong \Delta DEF$.	
Now s'pose $AB \neq DE$, $BC \neq EF$, and $AC \neq DF$.	
Without loss of generality, assume $AB > DE$ and $AC > DF$.	
Choose a point B' on $\overline{AB}$ such that $AB' = DE$ and choose a point C' on $\overline{AC}$ such that $AC' = DF$. Then $\square BCC'B'$ is convex.	
Then $\Delta AB'C' \cong \Delta DEF$.	
So $\angle AB'C' \cong \angle ABC$ and $\angle AC'B' \cong \angle ACB$.	
$\angle BB'C'$ is the supplement of $\angle AB'C'$ and $\angle CC'B'$ is the supplement of $\angle AC'B'$.	
$\sigma(\square BCC'B') = 360^\circ$	
But this is a contradiction, so $\Delta ABC \cong \Delta DEF$	

5. Show that if ΔABC is a triangle labeled in the standard way, and $a^2 + b^2 = c^2$, then $\angle BCA$ is a right angle.