Examlet 3 Advanced Geometry 4/10/15

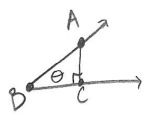
1. a) State the Fundamental Theorem on Similar Triangles.

If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{AC} = \frac{DE}{DF}$.

Good

10

b) State the definition of $\cos\theta$ for an acute angle θ.



Let the vertex of θ be B, and let A be a point on one of the rays that make θ. Drop a perpendicular from A to the other ray and call the foot C. Then $\cos\theta = \frac{BC}{BA}$.

Good.

c) Let ΔABC be a triangle. State the definition of the associated triangular region.

▲ABC is the union of $\triangle ABC$ with all of the internal points for $\triangle ABC$.

Good.

2. a) State the Neutral Area Postulate.

Associated with each polygonal region R is a nonnegative number $\alpha(R)$ called the area of R such that the following conditions are satisfied.

1) Congruence: If two triangles are congruent, then the associated triangular regions of the triangles have the same area.

2) Additivity: If R is the union of two nonoverlapping polygonal regions R_1 and R_2, then $\alpha(R) = \alpha(R_1) + \alpha(R_2)$. Good

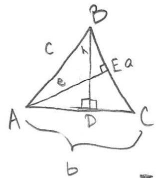
b) State the Euclidean Area Postulate.

If $\square ABCD$ is a rectangle, then $\alpha(\square ABCD) = AB \cdot BC$.

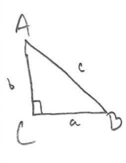
Great

3. State and prove the Law of Sines.

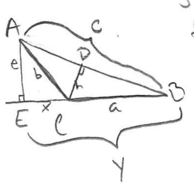
For any $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



Let $\triangle ABC$ be a triangle (hypothesis). $\triangle ABC$ may either be acute, right, or obtuse. Assume $\triangle ABC$ is an acute $\triangle$. (hypothesis) Drop a perpendicular from B to $\overleftrightarrow{AC}$ and call the foot D. (existence and uniqueness of $\perp$). D is on $\overline{AC}$ by a previous theorem (crossbar). Assign h to the perpendicular in this picture. So $\sin A = h/c$ $\sin C = h/a$ (algebra). $c\sin A = a\sin C$ (algebra). $\frac{\sin A}{a} = \frac{\sin C}{c}$ (algebra). Drop a perpendicular from A to $\overleftrightarrow{BC}$, call it e and call the foot E (existence and uniqueness of perpendiculars). E is on $\overline{BC}$ by the same previous theorem (crossbar). $\sin C = e/b$ and $\sin B = e/c$ so $e = b\sin C$ and $e = c\sin B$ (algebra) so $b\sin C = c\sin B$ so $\frac{\sin C}{c} = \frac{\sin B}{b}$ (algebra), so if $\triangle ABC$ is acute, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



Let $\triangle ABC$ be right. (hypothesis). Let C be the right angle without loss of generality (hypothesis). $\sin A = a/c$, $\sin B = b/c$, so $\frac{\sin A}{a} = \frac{1}{c} = \frac{\sin B}{b}$ (algebra). but $\sin C = \sin 90° = 1$, so $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (algebra). Let $\triangle ABC$ be an obtuse triangle as shown in the picture without loss of generality (hypothesis). Drop a perpendicular from C to $\overleftrightarrow{AB}$ and call it h and call the foot D (existence and uniqueness of $\perp$). D is on $\overline{AB}$ (crossbar). So $\sin B = h/a$ and $\sin A = h/b$ so $a\sin B = b\sin A$ (algebra). $\frac{\sin B}{b} = \frac{\sin A}{a}$ (algebra). Drop a perpendicular from A to $\overleftrightarrow{BC}$ call it e and call the foot E. (existence and uniqueness of perpendiculars). Call $\overline{EC}$ x and $\overline{EB}$ y. $\sin B = e/c$ and $\sin\angle ECA = e/b$ (algebra). $\angle ECA$ and $\angle ACB$ are supplements, so $\sin\angle ECA = \sin\angle ACB = \sin C$ (common sense). so $c\sin B = b\sin C$ (algebra). So $\frac{\sin B}{b} = \frac{\sin C}{c}$ so $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (algebra). □



W

Great

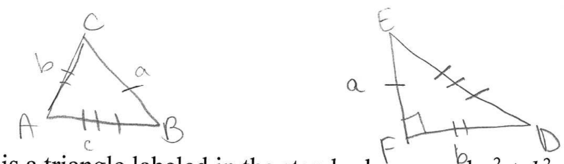
4. a) State a theorem (of your choice) from §6.2 about common perpendiculars.

If lines l and m admit a common perpendicular, then that common perpendicular is unique.

b) Provide good justifications in the blanks below for the corresponding statements:

Proposition: In hyperbolic geometry, if $\Delta ABC \sim \Delta DEF$, then $\Delta ABC \cong \Delta DEF$.

Statement:	Reason:
Let ΔABC and ΔDEF be two triangles such that $\Delta ABC \sim \Delta DEF$.	Hypothesis
If any one side of ΔABC is congruent to the corresponding side of ΔDEF, then $\Delta ABC \cong \Delta DEF$.	ASA
Now s'pose $AB \neq DE$, $BC \neq EF$, and $AC \neq DF$.	RAA hypothesis
Without loss of generality, assume $AB > DE$ and $AC > DF$.	Pigeonhole principle or, one is not enough
Choose a point B' on $\overline{AB}$ such that $AB' = DE$ and choose a point C' on $\overline{AC}$ such that $AC' = DF$. Then $\square BCC'B'$ is convex.	Point Construction Postulate and Theorem 4.6.7
Then $\Delta AB'C' \cong \Delta DEF$.	SAS
So $\angle AB'C' \cong \angle ABC$ and $\angle AC'B' \cong \angle ACB$.	Definition of Similar Triangles
$\angle BB'C'$ is the supplement of $\angle AB'C'$ and $\angle CC'B'$ is the supplement of $\angle AC'B'$.	Linear Pair
$\sigma(\square BCC'B') = 360°$	Algebra
But this is a contradiction, so $\Delta ABC \cong \Delta DEF$	Theorem 6.1.3



5. Show that if ΔABC is a triangle labeled in the standard way, and $a^2 + b^2 = c^2$, then $\angle BCA$ is a right angle.

Let ΔABC be a triangle such that $a^2+b^2=c^2$. (hyp).

Let ΔDEF be a triangle such that $BC=EF$, $AC=DF$, and $\angle EFD$ is a right angle. (Ruler Postulate, Protractor Postulate)

Then $a^2+b^2=ED^2$ (Pythagorean Theorem)

$ED^2=c^2$ (algebra)

$ED=c$ (algebra)

Great

Then $\Delta ABC \cong \Delta DEF$ (SSS)

$\angle BCA \cong \angle EFD$ (def of congruent triangles)

$\angle EFD$ is a right angle $\therefore$ $\angle BCA$ is a right angle (def of congruent triangle) $\square$