Exam 2 Calc 2 2/19/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Parts

1. Evaluate $\int x \cos x \, dx$.

$u = x \quad v = \sin x$

$u' = 1 \quad v' = \cos x$

$u \cdot v - \int u' \cdot v \, dx$

$(x)(\sin x) - \int (1)(\sin x) dx$

$x \sin x - \int \sin x \, dx$

$x \sin x + + \cos x + C$

$\boxed{x \sin x + \cos x + C}$

Great!

2. Give the form for a partial fractions decomposition of $\int \frac{x^2 - 4}{(x-5)^3 (x^2+1)} dx$, or explain why one does not exist.

$$\int \frac{x^2 - 4}{(x-5)^3 (x^2+1)} = \int \frac{A}{(x-5)} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx + E}{(x^2+1)} \, dx$$

Yes

3. What trigonometric substitution is a good idea for evaluating $\int \frac{1}{\sqrt{x^2+9}}\,dx$? [No, you don't need to work it out – just name the good substitution.]

3 tan θ

Yes!

because $\tan^2\theta + 1 = \sec^2\theta$

10

4. Evaluate $\int \tan^6\theta \sec^2\theta \, d\theta$.

$\int \tan^6\theta \sec^2\theta \, d\theta$

let $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$\frac{du}{\sec^2\theta} = d\theta$

$\int u^6 \cancel{\sec^2\theta} \frac{du}{\cancel{\sec^2\theta}}$

$\int u^6 \, du$

$\frac{u^7}{7} + C$

Great

$\boxed{\frac{\tan^7\theta}{7} + C}$

5. Evaluate $\int_{8}^{-8} \frac{1}{|x|^{2/3}}\,dx$

$$\underline{\lim_{a \to 0}} \int_{0}^{-8} \frac{1}{|x|^{2/3}}\,dx + \underline{\lim_{b \to 0}} \int_{8}^{0} \frac{1}{|x|^{2/3}}\,dx$$

$$\lim_{a \to 0} \int_{0}^{-8} x^{-2/3}\,dx + \lim_{b \to 0} \int_{8}^{0} x^{-2/3}\,dx$$

$$3x^{1/3} \Big|_{0}^{-8} + 3x^{1/3} \Big|_{8}^{0}$$

$$\left[3(-8)^{1/3} - 3(0)^{1/3}\right] + \left[3(0)^{1/3} - 3(8)^{1/3}\right]$$

$$-6 + -6$$

$$\boxed{= -12}$$

Excellent!

6. Derive the reduction formula $\int \sin^n\theta\, d\theta = -\frac{1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\, d\theta$.

$$\int \sin^n \theta\, d\theta = \frac{-1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\, d\theta$$

$\int \sin^n \theta\, d\theta$

$u = \sin^{n-1}\theta$ $\quad v = -\cos\theta$

$u' = n-1 \sin^{n-2}\theta\cos\theta$ $\quad v' = \sin\theta$

$= \int \sin^{n-1}\theta \sin\theta\, d\theta$

$= -\sin^{n-1}\theta\cos\theta - \int (n-1)\sin^{n-2}\theta\cos\theta(-\cos\theta)\, d\theta$

$= -\sin^{n-1}\theta\cos\theta + \int (n-1)\sin^{n-2}\theta\cos^2\theta\, d\theta$

$= -\sin^{n-1}\theta\cos\theta + \int (n-1)\sin^{n-2}\theta(1-\sin^2\theta)\, d\theta$

$= -\sin^{n-1}\theta\cos\theta + (n-1)\int \sin^{n-2}\theta - (n-1)\int \sin^n\theta\, d\theta$

$\int \sin^n\theta\, d\theta + (n-1)\int \sin^n\theta\, d\theta = -\sin^{n-1}\theta\cos\theta + (n-1)\int \sin^{n-2}\theta\, d\theta$

$n\int \sin^n\theta\, d\theta = -\sin^{n-1}\theta\cos\theta + (n-1)\int \sin^{n-2}\theta\, d\theta$

$$\boxed{\int \sin^n\theta\, d\theta = \frac{-1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{-n}\int \sin^{n-2}\theta}$$

Nice

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod! These integration technics are soooo hard! There was this one problem where I tried, like, a substitution, but the tutor Daddy hired for me said that wasn't good because there wasn't an x in the numerator. How can that even be, that a problem is easier if there's an extra x in it?"

Help Bunny by giving a good example of an integral which is easier if the function has an x included in the numerator, and explaining why that makes it easier.

well lets say your taking the Integral of the function:

$$\int \frac{x}{x^2+3}\,dx$$

you can do a usub BECAUSE of the x in the numerator. u will be x^2+3 and $\frac{1}{2}du = x\,dx$. B/c of the extra x you can completely avoid trig and everything involving trig. And bunny usub is something we learned in calc 1, we know how to do that! Hope this helps! :)

Wonderful.

8. Evaluate $\int \frac{(x^2+11x)\,dx}{(x-1)(x+1)^2}$. $= \int \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

clear denominators by multiplying all by $(x-1)(x+1)^2$

$$x^2+11x = (x+1)^2A + (x-1)(x+1)B + (x-1)C$$

when $x=1$: $1^2+11(1) = (1+1)^2A + (1-1)\cancel{(1+1)}B + \cancel{(1-1)}C$

$12 = 4A$

$A=3$

When $x=-1$: $(-1)^2 + 11(-1) = \cancel{(-1+1)^2}A + (-1-1)\cancel{(-1+1)}B + (-1-1)C$

$-10 = -2C$

$C=5$

Plug in $A=3$ and $C=5$, Solve for $B=-2$

$$\int \frac{3}{x-1} - \int \frac{2}{x+1} + \int \frac{5}{(x+1)^2}$$

$$3\ln|x-1| - 2\ln|x+1| - 5(x+1)^{-1} + C$$

$\int \frac{5}{(x+1)^2}dx$ $u=x+1$ $du=dx$

$\int 5u^{-2}du$

$\frac{5}{-3}u^{-1}+C$

$\frac{5}{-3}(x+1)^{-1}+C$

Excellent!

9. Set up an integral and evaluate it to find the volume of the solid resulting from rotating the region between $y = \sin x$ and the x-axis on the interval $[0, \pi]$ around the x-axis.

$$\pi \int_0^{\pi} \sin^2 x \, dx$$

Using reduction from #6

$$\pi \left(-\tfrac{1}{2} \sin x \cos x + \tfrac{1}{2} \int 1 \, dx \right)$$

$$\pi \left(-\tfrac{1}{2} \sin x \cos x + \tfrac{1}{2} x \right) \Big|_0^{\pi}$$

Well done!

$$\pi \left(-\tfrac{1}{2} \sin(\pi) \cos(\pi) + \tfrac{1}{2} \pi \right) - \pi \left(-\tfrac{1}{2} \sin(0) \cos(0) + \tfrac{1}{2}(0) \right) =$$

$$\pi \left(0 + \tfrac{1}{2} \pi \right) - \cancel{\pi (0)} = \boxed{\tfrac{1}{2} \pi^2}$$

10. Evaluate $\displaystyle\int \frac{x^2}{\sqrt{a^2 - x^2}} \, dx$. [Hint: If it's too daunting with the a, do it with a 1 there.]

$$\int \frac{a^3 \sin^2 \theta \cos \theta \, d\theta}{\sqrt{a^2 - a^2 \sin^2 \theta}}$$

$x = a \sin \theta$

$dx = a \cos \theta \, d\theta$

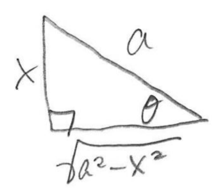
$$a^2 \int \frac{\sin^2 \theta \cos \theta \, d\theta}{\sqrt{\cos^2 \theta}}$$

$\sin \theta = \left(\frac{x}{a} \right)$

$\theta = \sin^{-1} \left(\frac{x}{a} \right)$

$$a^2 \int \sin^2 \theta \, d\theta$$

Reduction formula



$\cos \theta = \frac{\sqrt{a^2 - x^2}}{a}$

$$a^2 \left(-\tfrac{1}{2} \sin \theta \cos \theta + \tfrac{1}{2} \int 1 \, d\theta \right)$$

$$\frac{-a^2}{2} \sin \theta \cos \theta + \frac{a^2}{2} \theta$$

$$\boxed{\frac{-a^2}{2} \left(\frac{x}{a} \right) \left(\frac{\sqrt{a^2 - x^2}}{a} \right) + \frac{a^2}{2} \left(\sin^{-1} \left(\frac{x}{a} \right) \right) + C}$$

Excellent.