Exam 2 Differential Equations 3/4/16

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Find a solution to the system of differential equations

$$\frac{dx}{dt} = 3x$$
$$\frac{dy}{dt} = -2y$$

2. Determine whether $x(t) = 4e^{4t}$, $y(t) = e^{4t}$ is a solution to the system

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = x + 0y$$

3. Construct a system of differential equations, with all coefficients representing positive constants, to model the interaction of two populations where:
   - The first population would experience exponential growth in the absence of the second
   - Interaction between the two populations hurts the first population
   - The second population would experience logistic growth in the absence of the first
   - Interaction between the two populations benefits the second population

4. Find all equilibria of the system of differential equations:

$$\frac{dr}{dt} = 0.01r(25 - r) - 0.04rm$$

$$\frac{dy}{dt} = -0.2m + 0.02rm$$

5. Consider the system

$$\frac{dS}{dt} = -0.1SI$$
$$\frac{dI}{dt} = 0.1SI - 0.5I \ .$$
$$\frac{dR}{dt} = 0.25I$$

Use Euler's method with a step size of $\Delta t = 0.1$ to project $S(0.1)$, $I(0.1)$, and $R(0.1)$ if $S(0) = 60$, $I(0) = 4$, and $R(0) = 0$.

6. Consider the equation

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

for the motion of a simple harmonic oscillator. Consider the function $y(t) = \cos \beta t$. Under what conditions on β is $y(t)$ a solution?

7. Suppose $y(t)$ is a solution to the differential equation

$$\frac{d^2y}{dt^2} + \alpha \frac{dy}{dt} + \beta y = 0 .$$

What can you say about $k \cdot y(t)$, where k is a constant?

8. Find a non-trivial solution to the system

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = 5x.$$

9. Find a general solution to the system

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = -4y.$$

10. Find a general solution to the system

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = \alpha y.$$