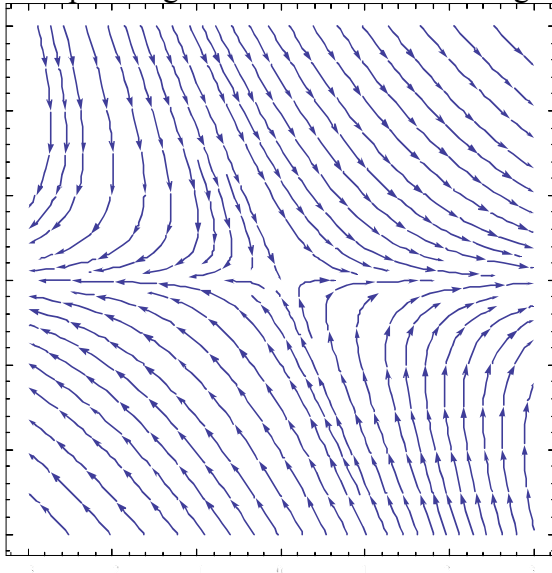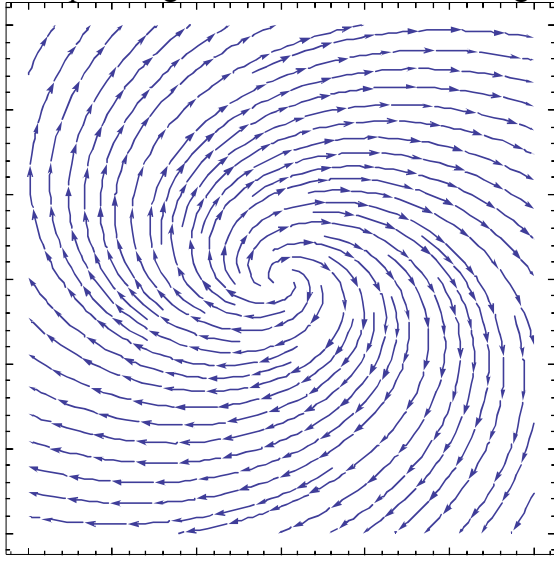**Exam 3   Differential Equations   4/8/16**

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Does the planar system whose phase plane is shown below have two real eigenvalues, or two complex eigenvalues? What are the signs on the real portions of the eigenvalues?

2. Estimate two (non-parallel) eigenvectors of the planar system whose phase plane is shown above.

3. Does the planar system whose phase plane is shown below have two real eigenvalues, or two complex eigenvalues? What are the signs on the real portions of the eigenvalues?

4. Suppose we have a planar system for which $\mathbf{Y}(t) = 2e^{3t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + 5e^{-t}\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ is a solution.

Find a solution passing through the point (1,0).

5. Is $y = t\,e^{\lambda t}$ a solution to the differential equation $y'' - 2\lambda\,y' + \lambda^2 y = 0$?

6. Consider the system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 6 \\ 1 & -2 \end{pmatrix} \mathbf{Y}$. Find a general solution to this system.

7. State and prove the Bandicoot Theorem.

8. Consider the system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} 6 & -5 \\ 5 & -4 \end{pmatrix} \mathbf{Y}$. Find a solution to this system satisfying the initial condition $\mathbf{Y}(0) = (0,1)$.

9. Consider the system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & b \\ 1 & -3 \end{pmatrix}\mathbf{Y}$. For what value(s) of b will solutions form closed curves, i.e. return to the initial condition after some amount of time?

10. Find the general solution to the linear system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & 9 \\ -2 & 4 \end{pmatrix} \mathbf{Y}$.