Examlet 2 Foundations of Advanced Math 2/26/16

1. Let $A = \{a, b, c\}$ and let $B = \{b, c\}$.

   a) True or False: $a \in A$

   True

   b) True or False: $A \in B$ the set A is not an element of set B

   False

   c) True or False: $B \in A$ the elements of B are elements of A, but B itself is not an element of A

   False

   d) True or False: $a \subseteq A$ a is not a set, so it can't be a subset

   False

   e) True or False: $\{a\} \in A$ a is an element of A, but the set containing a is not an element of A

   False

   f) True or False: $\{a\} \subseteq A$

   True

   g) True or False: $a \in \mathcal{P}(A)$ P(A) is the set of all the subsets of A, and a is not a set, so it is not an element of P(A)

   False

   h) True or False: $a \subseteq \mathcal{P}(A)$ a is not a set, so it can't be a subset

   False

   i) True or False: $B \in \mathcal{P}(A)$

   True

   j) True or False: $\mathcal{P}(B) \subseteq \mathcal{P}(A)$

   True

Great!

2. a) If $A \subseteq B$ and $A \subseteq C$, then $A \cup B \subseteq C$.

False,

$A=\{0\}$ $B=\{0,2\}$ $C=\{0,3\}$

$A \cup B=\{0,2\}$ Good.

$2 \in A \cup B$, but $2 \notin C$, so $A \cup B \nsubseteq C$. $\square$

b) If $A \subseteq B$ and $A \subseteq C$, then $A \cap B \subseteq C$.

Take $x \in A \cap B$, so $x \in A$ and $x \in B$. If $x \in A$, then $x \in C$ because $A \subseteq C$, so $A \cap B \subseteq C$. $\square$

Excellent!

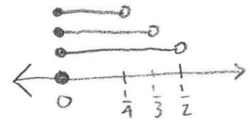
3. For each $n \in \mathbb{N}$, let $A_n = \left[0, \frac{1}{n+1}\right)$.

a) What is $\bigcap_{n\in\{1,2,3\}} A_n$?

$A_1 = [0, \frac{1}{2})$

$A_2 = [0, \frac{1}{3})$

$A_3 = [0, \frac{1}{4})$



$\bigcap_{n\in\{1,2,3\}} A_n = \underline{[0, \frac{1}{4})}$

b) What is $\bigcup_{n\in\{1,2,3\}} A_n$?

$A_1 = [0, \frac{1}{2})$

$A_2 = [0, \frac{1}{3})$

$A_3 = [0, \frac{1}{4})$

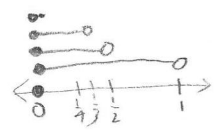
$\bigcup_{n\in\{1,2,3\}} A_n = \underline{[0, \frac{1}{2})}$

c) What is $\bigcap_{n\in\mathbb{N}} A_n$?

$A_0 = [0, 1)$

$A_1 = [0, \frac{1}{2})$

$A_2 = [0, \frac{1}{3})$



there is no rational number closest to zero, so only zero is in the intersection.

$\bigcap_{n\in\mathbb{N}} A_n = \underline{\{0\}}$

Excellent!

10

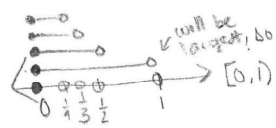
d) What is $\bigcup_{n\in\mathbb{N}} A_n$?

$n = 0$

$A_0 = [0, \frac{1}{0+1}) = [0, 1)$

$A_1 = [0, \frac{1}{2})$

$A_2 = [0, \frac{1}{3})$



$\bigcup_{n\in\mathbb{N}} A_n = \underline{[0, 1)}$

4. $\forall x \in \mathbb{R}, |x| \geq 0$.

First case: $x \geq 0$, so by definition $|x| = x$. So $|x| \geq 0$.

Second case: $x < 0$, so by definition $|x| = -x$, so $-|x| = x$ and $-|x| < 0$. Then we can add $|x|$ to both sides by the CAP and we get $0 < |x|$ so we know $|x| \geq 0$.

In all cases $|x| \geq 0$, so it is true $\forall x \in \mathbb{R}$. $\square$

Excellent!

5. Suppose $r \in \mathbb{R}$, and $r \geq 1$. Then $r^n \geq 1$ for all $n \in \mathbb{N}$.

So we will proceed by induction to prove if $r \in \mathbb{R}$ and $r \geq 1$, then $r^n \geq 1$ for all $n \in \mathbb{N}$. First we will start out with a base case, $n=0$. So, $r^0 \geq 1$ which is true because $1=1$, so $1 \geq 1$. For our second base case, $n=1$, so $r^1 \geq 1$ which is true because $r \geq 1$. Now we will suppose the statement is true for $n=K$ so that $r^K \geq 1$ and then we need to prove that $r^{K+1} \geq 1$. So we know that $r \geq 1$ which means $r > 0$ because $1 > 0$, so we can use the CMP to get $r^K \cdot r \geq 1 \cdot r$ so $r^{K+1} \geq r$. Since $r \geq 1$, by the TPI, $r^{K+1} \geq 1$. So by induction, $r^n \geq 1$ for all $n \in \mathbb{N}$. $\square$

Great