Examlet 4 Foundations of Advanced Math 4/15/16

1. a) State the definition of a relation from A to B.

b) State the definition of the degree of a vertex v in a graph.

c) State the definition of a tree.

2. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $a \sim b$ iff $5|(a - b)$. Show that $\sim$ is an equivalence relation, being clear about your reasoning.

3. a) Express the definition of the sum of two functions function formally in terms of ordered pairs.

b) Express the definition of the composition of two functions formally in terms of ordered pairs.

4. a) Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a reflexive relation.

b) Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a symmetric relation.

c) Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a transitive relation.

5. Let G be a graph and say two vertices u and v of G are related iff u and v are joined by a walk of odd length. Is this relation reflexive? symmetric? transitive? Support your answers well.