Examlet 2a **Foundations of Advanced Math** **2/24/17**

1. (a) What is $\{1,2\} \cap \{2,3\}$?

$\{2\}$ intersection: things in both sets.

(b) What is $(1,2) \cap (2,3)$?

$\emptyset$ 2 is not included in either set since its intervals.

(c) What is $[1,2] \cap [2,3]$?

$\{2\}$ 2 is the only thing in both sets.

(d) What is $\{1,2\} \cup \{2,3\}$?

$\{1,2,3\}$

(e) What is $(1,2) \cup (2,3)$?

$(1,2)\cup(2,3)$

Good attempt to catch people out, but 2 is not included in the intervals. So you cant include it. I cant think of a better way to write it though.

(f) What is $[1,2] \cup [2,3]$?

$[1,3]$ as 2 is included, everything is fine.

(g) What is $\{1,2\} - \{2,3\}$?

$\{1\}$ set difference: take out the stuff in set 2 thats in set 1.

(h) What is $(1,2) - (2,3)$?

$(1,2)$ as the two intervals share nothing in common, this is the difference.

(i) What is $[1,2] - [2,3]$?

$[1,2)$ Two can no longer be included as it is in the other interval

(j) What is $\mathcal{P}\{1,2\}$?

$\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

Great!

as we have 2^2 elements in the power set, were good.

2. (a) State the definition of

$$\bigcap_{i \in I} A_i$$

$\{x \mid x \in A_i \text{ for all } i \in I\}$

(b) Let $\mathbb{Z}^+ = \{n | n \in \mathbb{Z}^+ n > 0\}$. If $A_n = \left(\frac{1}{n}, 1\right) \forall n \in \mathbb{Z}^+$, what is

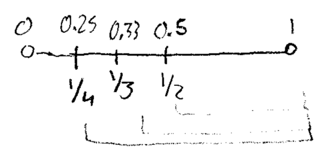
$\{n \mid n \geq 2 \text{ and } n \in \mathbb{Z}\}$ $$\bigcap_{n \in \mathbb{Z}^+} A_n$$

$A_2 = \left(\frac{1}{2}, 1\right)$

$A_3 = \left(\frac{1}{3}, 1\right)$

$A_4 = (1/4, 1)$

$$\bigcap_{n \in \mathbb{Z}^+} A_n = \left(\frac{1}{2}, 1\right)$$



(c) Let $\mathbb{Z}^+ = \{n | n \in \mathbb{Z}^+ n > 0\}$. If $A_n = \left(\frac{1}{n}, 1\right) \forall n \in \mathbb{Z}^+$, what is

$$\bigcup_{n \in \mathbb{Z}^+} A_n$$

$$\bigcup_{n \in \mathbb{Z}^+} A_n = (0, 1)$$

Zero is not in any of the sets, but the sets keep including more and more and get infinitely close to zero.

Great

3. $(A \cup B)' = A' \cap B'$

FIRST LETS TAKE $X \in (A \cup B)'$, THIS CAN BE WRITTEN IN LOGIC NOTATION AS $\neg(x \in A \vee x \in B)$, THROUGH DEMORGANS LAW WE KNOW THAT $\neg(x \in A \vee x \in B)$ IS LOGICALLY EQUIVALENT TO $\neg x \in A \wedge \neg x \in B$ REWRITING THIS IN SET NOTATION GIVES US $X \in A' \cap B'$, SO

$$(A \cup B)' \subseteq A' \cap B'$$

NOW LETS TAKE $X \in A' \cap B'$ THIS IMPLIES $\neg X \in A \wedge \neg X \in B$, THROUGH DEMORGANS LAW WE KNOW THAT $\neg X \in A \wedge \neg X \in B$ IS LOGICALLY EQUIVALENT TO $\neg(X \in A \vee X \in B)$ REWRITING THIS IN SET NOTATION GIVES US $X \in (A \cup B)'$, SO $A' \cap B' \subseteq (A \cup B)'$, SO BY MUTUAL INCLUSION

$$(A \cup B)' = A' \cap B' \ \square$$

Great

4.

$$A \cap \bigcup_{i\in I} B_i = \bigcup_{i\in I}(A \cap B_i)$$

FIRST LETS TAKE $X \in A \cap \bigcup_{i\in I} B_i$. THIS CAN BE WRITTEN IN LOGIC NOTATION AS $X \in A \wedge \exists i \in I, X \in B_i$, NOW, SINCE $X \in A$ CANNOT BE AFFECTED BY THE THERE EXISTS STATEMENT BECAUSE IT IS NOT AN INDEXING SET, WE CAN REWRITE THIS AS $\exists i \in I, (X \in A \wedge X \in B_i)$. THIS REWRITTEN IN SET NOTATION IS $X \in \bigcup_{i\in I}(A \cup B_i)$, SO $A \cap \bigcup_{i\in I} B_i \subseteq \bigcup_{i\in I}(A \cap B_i)$

NOW LETS TAKE $X \in \bigcup_{i\in I}(A \cup B_i)$. THIS CAN BE REWRITTEN IN LOGIC NOTATION AS $\exists i \in I, (X \in A \wedge X \in B_i)$, NOW, SINCE $X \in A$ CANNOT BE AFFECTED BY THE THERE EXISTS STATEMENT DUE TO ITS STATUS NOT AS AN INDEXING SET WE CAN REWRITE THIS AS $X \in A \wedge \exists i \in I, X \in B_i$. REWRITING THIS IN SET NOTATION GIVES US $X \in A \cap \bigcup_{i\in I} B_i$, SO $\bigcup_{i\in I}(A \cap B_i) \subseteq A \cap \bigcup_{i\in I} B_i$ AND BY MUTUAL INCLUSION:

$$A \cap \bigcup_{i\in I} B_i = \bigcup_{i\in I}(A \cap B_i) \quad \square$$

Well done!

5. (a) If $a > 0$ and $b > 0$, then $a + b > 0$.

CAP to add b to both sides of $a > 0$ to get $a + b > b$ because we know $b > 0$ we can set up the inequality $a + b > b > 0$. By the transitive property $a + b > 0$ $\square$

Good

(b) If $a < 0$ and $b < 0$, then $a \cdot b > 0$.

Use the CAP to add $-a$ to both sides of $a < 0$ to get $0 < -a$. multiply both sides of $b < 0$ by $-a$ using CMP to get $-ab < 0$. Add $(a \cdot b)$ to both sides using the CAP to get.

$a \cdot b - a \cdot b < ab$. This simplifies to $0 < a \cdot b$ which is the same as $a \cdot b > 0$ $\square$

Nice!