Examlet 4b **Foundations of Advanced Math** **4/14/17**

1. (a) State the definition of a relation from A to B.

   (b) Give an example of a relation from $\{1, 2, 3, 4\}$ to $\{1, 2, 3, 4\}$ which is reflexive and symmetric, but is not transitive.

2. Which of the following are partitions of $S = \{a, b, c, d, e\}$? Mark all which are.

   - ☐ $\{\{a, b, c\}, \{d, e\}\}$
   - ☐ $\{\{a, b, d\}, \{c\}\}$
   - ☐ $\{\{a, b\}, \{c, d, e\}, \{\}\}$
   - ☐ $\{a, c, d\}, \{b, e\}$
   - ☐ $\{\{a, b\}, \{b, c\}, \{c, d\}, \{e\}\}$

3. Express the definition of a surjective function in terms of ordered pairs.

4. Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then $\sim$ is a reflexive relation.

5. (a) State the definition of a graph.

(b) Suppose G is a graph with every vertex having degree at least 1. Create a relation $\sim$ on the vertices of G by saying that two vertices v_1, v_2 of G are related iff there exists a walk from v_1 to v_2 which has no edge used more than once. Is $\sim$ reflexive? Symmetric? Transitive?