Exam 3 **Calculus 2** **3/30/18**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Find the first 3 partial sums of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

$S_1 = \frac{(-1)^1}{1!} = \underline{-1}$

$S_2 = -1 + \frac{(-1)^2}{2!} = -1 + \frac{1}{2\times1} = \underline{-\frac{1}{2}}$

$S_3 = -\frac{1}{2} + \frac{(-1)^3}{3!} = -\frac{1}{2} - \frac{1}{6} = \underline{-\frac{2}{3}}$

Great

2. Find the sum of the geometric series

$$\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \frac{8}{81} + \ldots$$

$\underline{a = \frac{1}{3}}$

$\underline{r = -\frac{2}{3}}$

$\because |r| = |-\frac{2}{3}| < 1$

Great

$\therefore$ By geometric series sum:

$S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-(-\frac{2}{3})} = \frac{\frac{1}{3}}{\frac{5}{3}} = \underline{\frac{1}{5}}$

3. Find the 6th degree MacLaurin polynomial for $f(x) = \cos x$.

0 $f(x) = \cos x$ $f(0) = 1$

1 $f'(x) = -\sin x$ $f'(0) = 0$

2 $f''(x) = -\cos x$ $f''(0) = -1$

3 $f'''(x) = \sin x$ $f'''(0) = 0$

4 $f^{(4)}(x) = \cos x$ $f^{(4)}(0) = 1$

5 $f^{(5)}(x) = -\sin x$ $f^{(5)}(0) = 0$

6 $f^{(6)}(x) = -\cos x$ $f^{(6)}(0) = -1$

$$p(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$\frac{1}{1} x^0 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6$$

$$\boxed{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}}$$

Yes

4. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ converges or diverges.

$\frac{1}{1+n^2}$ is similar to $\frac{1}{n^2}$ which I know converges because its a p-series with $p > 1$.

I know $1 > 0$

so $n^2 + 1 > n^2$

so $\frac{1}{n^2+1} < \frac{1}{n^2}$

By the comparison test, $\frac{1}{n^2+1}$ must converge because it is less than $\frac{1}{n^2}$ which converges!

Excellent!

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{1+5^n}{1+3^n}$ converges or diverges.

Since $\lim_{n\to\infty} \frac{1+5^n}{1+3^n} \neq 0$, this series diverges by the Test For Divergence.

6. Find the Taylor series of degree 3 centered at $x = 9$ for $f(x) = \sqrt{x}$. $\to x^{\frac{1}{2}}$

$$p(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\frac{f^{(0)}(9)}{0!}(x-9)^0 + \frac{f^{(1)}(9)}{1!}(x-9)^1 + \frac{f^{(2)}(9)}{2!}(x-9)^2 + \frac{f^{(3)}(9)}{3!}(x-9)^3$$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $\quad f'(9) = \frac{1}{6}$

$f''(x) = \frac{-1}{4}x^{-3/2}$ $\quad f''(9) = -\frac{1}{108}$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(9) = \frac{1}{648}$

$$1 + \frac{1}{6}(x-9) - \frac{\frac{1}{108}}{2}(x-9)^2 + \frac{\frac{1}{648}}{6}(x-9)^3$$

$$\boxed{1 + \frac{(x-9)}{6} - \frac{(x-9)^2}{216} + \frac{(x-9)^3}{3888}}$$

Great!

7. Biff is a Calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, Calc is so impossible! Every time I think I know how to do it, they tell me it's not that simple anymore. So I did the test for series, right? Like, for $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ I said Comparison test, right? Since I know it's less than $\sum_{n=1}^{\infty} \frac{1}{n^2}$, right, so it converges? But they said that was wrong because something stupid, so I think I'm just going to drop out and be homeless."

Help Biff out by explaining what might be wrong with his approach.

Because of the $(-1)^n$ this series alternates signs. To use the comparison test all terms have to be positive which they aren't in this case! You can use the Alternating Series test instead. You already know the signs alternate. The $\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$ so it passes that part of the test. The derivative of $\frac{1}{n^2}$ is $-2n^{-3}$ which is negative so it is decreasing. This means it converges by the AST. While this series does converge, you cannot use the Comparison test to prove so.

Excellent!

8. Find the radius of convergence of the power series

ratio test

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{5^n}$$

$\lim_{n\to\infty} \left| \frac{x^{(n+1)}}{5^{(n+1)}} \Big/ \frac{x^n}{5^n} \right|$ we can leave $(-1)^n$ out because $|(-1)^n|$ is always 1

$\lim_{n\to\infty} \left| \frac{x^{\cancel{n+1}}}{5^{\cancel{n+1}}} \cdot \frac{\cancel{5^n}}{\cancel{x^n}} \right|$ $\quad \lim_{n\to\infty} \left| \frac{x}{5} \right| = \left| \frac{x}{5} \right|$

if $\left| \frac{x}{5} \right| < 1$ then it converges so

$|x| < 5$ radius of convergence is 5

Excellent!

9. Use a power series with at least 4 nonzero terms to approximate

$$\int_0^{0.1} e^{(-x^2)}\,dx$$

we know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

Substitute $-x^2$ in for x

$1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$ integrate

$$\int_0^{0.1} 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6}$$

$$\left[x - \frac{1}{3}x^3 + \frac{1}{10}x^5 - \frac{1}{42}x^7 \right]_0^{0.1}$$

$$\left(0.1 - \frac{1}{3}(0.1)^3 + \frac{1}{10}(0.1)^5 - \frac{1}{42}(0.1)^7\right) - 0$$

$$\int_0^{0.1} e^{(-x^2)}\,dx \approx \boxed{0.0996676643}$$

Excellent!

$2(n+1)+1$

10. Determine the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \qquad \sum_{n=\emptyset}^{\infty} \frac{x^{2n} x^{1}}{2n+1}$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^{2n}} x^{\cancel{3}\,2}}{2n+3} * \frac{2n+1}{\cancel{x^{2n}} \cancel{x^{1}}} \right| = |x^2| \lim_{n\to\infty} \left| \frac{2n+1}{2n+3} \right| = 1$$

$x^2 < 1$

$x < 1$ = Radius

$-1 < x < 1$

$\boxed{\underline{\text{Interval:}}\ \underline{(-1,\ 1)}}$

<u>If $x = -1$</u>, $\sum_{n=\emptyset}^{\infty} \frac{(-1)}{2n+1} = -1 - \frac{1}{3} - \frac{1}{5} - \frac{1}{7}$

$\sum_{n=\emptyset}^{\infty} \frac{(-1)}{2n+1}$ <u>diverges to $-\infty$</u>

<u>Nice Job!</u>

<u>If $x = 1$</u> $\sum_{n=\emptyset}^{\infty} \frac{(1)^{2n}(1)^{1}}{2n+1} = \sum_{n=\emptyset}^{\infty} \frac{1}{2n+1}$

$u = 2x+1$
$du = 2dx$

$\int_0^{\infty} \frac{1}{2x+1}\,dx = \lim_{b\to\infty} \frac{1}{2}\ln|2x+1| \Big|_0^b = \infty$

By the <u>integral test</u>, since <u>$\lim_{b\to\infty} \int_0^b \frac{1}{2x+1}\,dx$ diverges</u>

So does $\sum_{n=\emptyset}^{\infty} \frac{1}{2n+1}$. So $x = 1$ is <u>not</u> included