Exam 4 **Calculus 2** **4/20/18**

Each problem is worth 10 points. For full credit provide good justification for your answers.

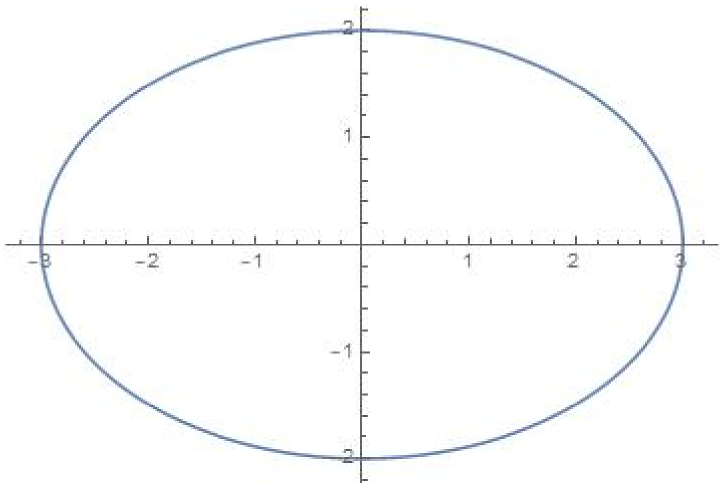
1. (a) Convert $(1, \frac{\pi}{2})$ from polar to rectangular coordinates.

   (b) Convert $(3, -3)$ from rectangular to polar coordinates.

2. (a) Convert $x^2 + y^2 = 25$ to an equation in polar coordinates.

   (b) Convert $\theta = \frac{\pi}{2}$ to an equation in rectangular coordinates.

3. Write an equation for the conic section shown below:



4. Set up an integral for the length of the curve $x = t - 2\sin t$, $y = 1 - 2\cos t$ for $0 \leq t \leq 4\pi$.

5. Find the exact coordinates of the leftmost point on the parametric curve with equations $x = t^4 - t^2$, $y = t + \ln(t)$.

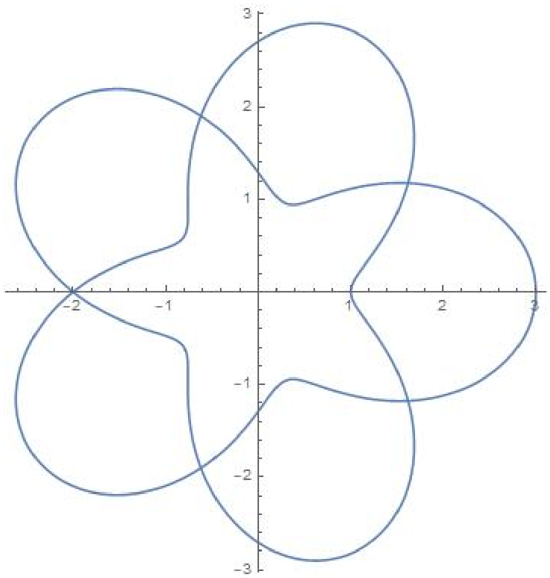
6. Identify the graph of $y^2 - 8y = 16x^2$ as a parabola, hyperbola, or ellipse, give coordinates of its vertices, and sketch a decent graph.

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, Calc is so impossible! Every time I think I know something, they tell me it's not that simple anymore. So there was this question on the quiz, right? And it said to give three different pairs of coordinates for this point, right? But that's crazy, because, I mean, like if the coordinates for a point are $(2, 2)$ then they just *are*, right? If you give different coordinates then it's a different point, right? But the TA said something about how it's because of polar stuff, which I really don't get because there's trig, you know?"

   Help Bunny out by explaining clearly what's going on.

8. Set up an integral or integrals for the area of the region enclosed by the loop of the curve with parametric equation $x = 7t - t^3$, $y = 8t^2$.

9. Set up an integral or integrals for the area of the region inside the curve with polar equation $r = 2 + \cos \frac{5\theta}{2}$.



10. [Anton 6th p. 712] In the late seventeenth century the Italian astronomer Giovanni Domenico Cassini (1625-1712) introduced the family of curves

$$(x^2 + y^2 + a^2)^2 - b^4 - 4a^2x^2 = 0 \quad (a > 0, b > 0)$$

in his studies of the relative motions of the Earth and the Sun.

(a) Show that if $a = b$, then the polar equation of the Cassini oval is $r^2 = 2a^2 \cos 2\theta$, which is a lemniscate.

(b) Show that in a polar coordinate system the distance d between the points (r_1, θ_1) and (r_2, θ_2) is

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)}$$

Extra Credit [5 points possible]: Show that the lemniscate from problem 10 is the curve traced by a point that moves in such a way that the product of its distances from the polar points $(a, 0)$ and (a, π) is a^2.