1. (a) What is $\{1,3\} \cap \{3,4\}$?

$\{3\}$

(b) What is $(1,3) \cap (3,4)$?

$\emptyset$

(c) What is $[1,3] \cap [3,4]$?

$\{3\}$

(d) What is $\{1,3\} \cup \{3,4\}$?

$\{1,3,4\}$

(e) What is $(1,3) \cup (3,4)$?

$(1,3) \cup (3,4)$

(f) What is $[1,3] \cup [3,4]$?

$[1,4]$

(g) What is $\{1,3\} - \{3,4\}$?

$\{1\}$

(h) What is $(1,3) - (3,4)$?

$(1,3)$

Great

(i) What is $[1,3] - [3,4]$?

$[1,3)$

(j) What is $\mathcal{P}\{1,3\}$?

$P\{1,3\} = \{\emptyset, \{1\}, \{3\}, \{1,3\}\}$

2. (a) State the definition of

$$\bigcap_{i \in I} A_i$$

$\{x \mid x \in A_i \text{ for all } i \in I\}$

(b) Let P be the set of positive real numbers. For each $x \in P$, let $A_x = [x, 2x]$. Find

$$\bigcap_{x \in P} A_x$$

$A_1 = [1,2]$

$A_2 = [2,4]$

$A_{12} = [12, 24]$

$\bigcap_{x \in P} A_x = \emptyset$ there is no common element in all A_x

(c) Let P be the set of positive real numbers. For each $x \in P$, let $A_x = [x, 2x]$. Find

$$\bigcup_{x \in P} A_x$$

Doesn't include zero, but does get infinitely close to zero.

$\bigcup_{x \in P} A_x = (0, \infty)$ Great

3. $(A \cap B)' = A' \cup B'$

Take $X \in (A \cap B)'$, so $X \notin A \cap B$, which we can write as $\neg(X \in A \wedge B)$, and by DeMorgan's law we know that is logically equivalent to $\neg X \in A \vee \neg X \in B$, or $X \notin A$ or $X \notin B$, or $X \in A'$ or $X \in B'$, so $X \in A' \cup B'$ $\therefore (A \cap B)' \subseteq A' \cup B'$.

Take $X \in A' \cup B'$, so $X \notin A \cup X \notin B$, which can be written as $\neg X \in A \vee \neg X \in B$, and by DeMorgan's law we know that is logically equivalent to $\neg(X \in A \wedge B)$, or $X \notin A \cap B$, so $X \in (A \cap B)'$, $\therefore$ $A' \cup B' \subseteq (A \cap B)'$.

Since they are both subsets of each other, $(A \cap B)' = A' \cup B'$ by definition of equal sets.

Excellent!

4. For any sets A, B, and C, $(B - A) \subseteq (C - A) \cup (B - C)$.

Take $x \in B - A$, so $x \in B$ and $x \notin A$. Now we consider two cases:

Case 1, $x \in C$: So we know $x \in C$ and $x \notin A$, which means $x \in C - A$. But that means $x \in C - A$ or $x \in B - C$ is true, so $x \in (C - A) \cup (B - C)$.

Case 2, $x \notin C$: So we know $x \notin C$ and $x \in B$ (from $x \in B - A$), which means $x \in B - C$. Then $x \in B - C$ or $x \in C - A$, so $x \in (C - A) \cup (B - C)$.

So in either case we have $x \in B - A \Rightarrow x \in (C - A) \cup (B - C)$, and thus $B - A \subseteq (C - A) \cup (B - C)$ as desired. $\square$

5. (a) If $0 < a$ and $a < b$, then $a^2 < b^2$.

$a > 0 \quad b > a$ so by TPI $b > 0$

If we use CMP we can multiply a to both side of $a < b$. so that $a^2 < ab$. Now if we do the same CMP but multiply by b, we get $a \cdot b < b^2$.

This gives us $a^2 < a \cdot b < b^2$

So by TPI, $a^2 < b^2$ $\square$

$a^2 < a \cdot b < b^2$

$b \cdot a < b \cdot b$

$ab < b^2$

$a \cdot a < b \cdot a$

$a^2 < a \cdot b$

Great

(b) $\forall x \in \mathbb{R}, |x| \geq 0$.

Def^n $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Case 1: $x \geq 0$ so that $x = |x|$. So since $|x| = x$ and $x \geq 0$, then $|x| \geq 0$.

Case 2: $x < 0$ so that $|x| = -x$, this is the same as $-|x| = x$. Since $x < 0$ $-|x| < 0$. However if we add $|x|$ to both sides (CAP) we get $0 < |x|$. Since $|x| > 0$, then $|x| \geq 0$.

Good

So by all cases, $|x| \geq 0$. $\square$