Examlet 2 **Foundations of Advanced Math** **3/1/19**

1. Write each of the sets below as simply as possible:

   (a) What is $\{1,2\} - \{2,4\}$?

   $\{1\}$

   (b) What is $(1,2) - [2,4]$?

   $(1,2)$

   (c) What is $[1,2] - [2,4]$?

   $[1,2)$

   (d) What is $\{1,2\} \cup \{2,4\}$?

   $\{1,2,4\}$

   (e) What is $(1,2) \cup [2,4]$?

   $(1,4]$

   (f) What is $[1,2] \cup (2,4)$?

   $[1,4)$

   (g) What is $\{1,2\} \cap \{2,4\}$?

   $\{2\}$

   (h) What is $[1,2] \cap [2,4]$?

   $\{2\}$

   Excellent!

   (i) What is $\{1,2\} \times \{2,4\}$?

   $\{(1,2),(1,4),(2,2),(2,4)\}$

   (j) What is $\mathcal{P}\{1,2\}$?

   $P\{1,2\} = \{\{1\},\{2\},\{1,2\},\emptyset\}$

2. $A \cap B \subseteq A \cup B$

Well, take $x \in A \cap B$, so $x \in A$ and $x \in B$.
Then since $x \in A$, we know it's true that $x \in A$ or $x \in B$.
So $x \in A \cup B$.
Thus since $x \in A \cap B \Rightarrow x \in A \cup B$, $A \cap B \subseteq A \cup B$. $\square$

3.

$$A \cup \bigcap_{i \in I} B_i = \bigcap_{i \in I} (A \cup B_i)$$

Take $x \in \left(A \cup \bigcap_{i \in I} B_i\right) \Longleftrightarrow x \in A \vee x \in \bigcap_{i \in I} B_i \Longleftrightarrow x \in A \vee$ $x \in B_i$ for all $i \in I \Longleftrightarrow x \in \bigcap_{i \in I} (A \cup B_i)$. So,
$A \cup \bigcap_{i \in I} B_i \subseteq \bigcap_{i \in I} (A \cup B_i)$ and $\bigcap_{i \in I} (A \cup B_i) \subseteq A \cup \bigcap_{i \in I} B_i$

$\therefore$ By mutual inclusion $A \cup \bigcap_{i \in I} B_i = \bigcap_{i \in I} (A \cup B_i)$. $\square$

Good

4. If $a, b, c \in \mathbb{R}$ with $a < b$ and $c < 0$, then $ac > bc$.

take $a<b$ and $c<0$. add $-c$ to both sides of $c<0$ by the comparison addition principle, so

$-c+c<0+(-c) \Rightarrow 0<-c$

then multiply both sides of $a<b$ by $-c$ by the comparison multiplacation principle, so

$a(-c)<b(-c) \Rightarrow -ac<-bc$.

add ac and bc to both sides by CAP

$ac-ac+bc<bc-bc+ac$

$bc<ac$, which is the same as

$ac>bc$, so the statement is true. □

Nice!

5. $\forall x, y, z \in \mathbb{R}$, $|x + y + z| \leq |x| + |y| + |z|$.

Take $|x+y+z|$.

Then, $|(x+y)+z| \leq |x+y| + |z|$ ← triangle inequality theorem

$|x+y| + |z| \leq |x| + |y| + |z|$

Therefore, since

$$|x+y+z| \leq |x+y| + |z| \leq |x| + |y| + |z|,$$

by Transitive Property of Inequalities,

$$|x+y+z| \leq |x| + |y| + |z|. \square$$

Good