Examlet 4 **Foundations of Advanced Math** **4/19/19**

1. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $a \sim b \Leftrightarrow a - b$ is odd.

   (a) Determine whether and why $\sim$ is reflexive.

   (b) Determine whether and why $\sim$ is symmetric.

   (c) Determine whether and why $\sim$ is transitive.

2. Consider the relation on some collection of sets defined by $A \approx B \Leftrightarrow \exists$ a bijection $f : A \to B$.

   (a) Determine whether and why $\approx$ is reflexive.

   (b) Determine whether and why $\approx$ is symmetric.

   (c) Determine whether and why $\approx$ is transitive.

3. Let $S = \{a, b, c, d\}$, and let $\sim = \{(a, a), (b, b), (b, c), (c, b), (c, c), (d, d)\}$.

   (a) Give the equivalence classes of $\sim$.

   (b) Give the partition associated with $\sim$.

4. Suppose that G is a graph with at least one cycle. We say that two vertices v_1 and v_2 of a graph G are **on a common cycle of** $G \Leftrightarrow \exists$ a cycle including v_1 and v_2.

   (a) The relation of being on a common cycle of a graph is reflexive.

   (b) The relation of being on a common cycle of a graph is symmetric.

   (c) The relation of being on a common cycle of a graph is transitive.

5. (a) Give all trees with $n \leq 5$ vertices.

(b) The minimum number of vertices with degree 1 in a tree with n vertices is

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