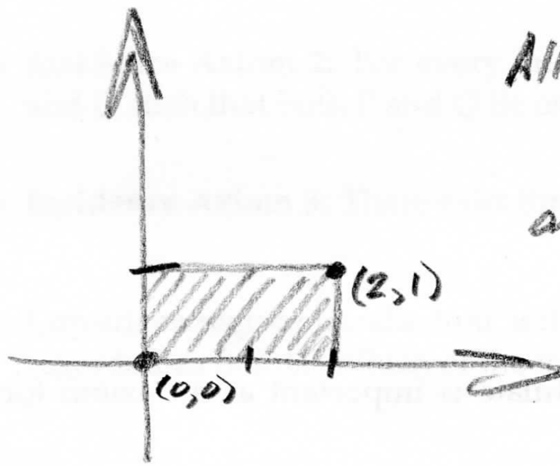
Examlet 1 **Advanced Geometry** **2/11/19**

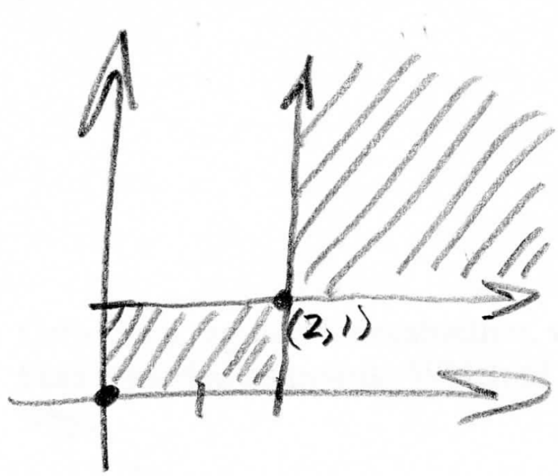
1. Let $A = (0, 0)$ and $B = (2, 1)$, and let ρ be the taxicab metric.

   a) Describe all points on $\overline{AB}$.



All points with $0 \le x \le 2$
and $0 \le y \le 1$
are between A and B.

   b) Describe all points on $\overrightarrow{AB}$.



2. a) State the SAS Postulate.

Let there be Two triangles ΔABC and ΔDEF.

Then if $\overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$, and $\overline{BC} \cong \overline{EF}$ then $\Delta ABC \cong \Delta DEF$

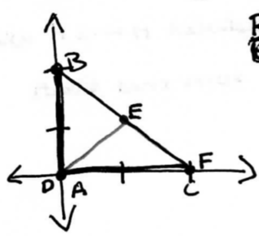
Good

b) Explain clearly why the SAS Postulate is important as an axiom for Neutral Geometry.

Well, you see there is another metric known as taxicab that has also satisfied the 5 previous axioms of neutral geometry but SAS fails in taxicab, making it necessary and independent of the other axioms.

Take two triangles ΔABC where $A = (0,0)$, $B = (0,2)$ and $C = (2,0)$ and ΔDEF where $D = (0,0)$, $B = (1,1)$ and $C = (2,0)$

then, if you look in the taxicab metric



$BA = 2$ and $DE = 2$

so $\overline{BA} \cong \overline{DE}$

$\angle BAC = 90° = \angle DEF$

so $\angle BAC \cong \angle DEF$

and $AC = 2 = EF$

so $\overline{AC} \cong \overline{EF}$

Excellent!

so (if SAS applied in taxicab) $\Delta ABC \cong \Delta DEF$, but we can clearly see $BC = 4$ while $DF = 2$ meaning they are not congruent, so SAS must be an independent axiom necessary to distinguish taxicab and neutral geometry.

3. Recall the three axioms of Incidence Geometry:

- **Incidence Axiom 1:** For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.

- **Incidence Axiom 2:** For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.

- **Incidence Axiom 3:** There exist three points that do not all lie on any one line.

a) Consider a regular tetrahedron, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why?

It satisfies axiom 2 only, because every face has three vertices, so clearly each line has two distinct points.

Okay.

b) Consider a cube, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why?

This satisfies axioms 2 & 3.
Again, every face has multiple vertices, satisfying axiom 2. Axiom 3 is exemplified in figure 1.3, as A, B, C are non-collinear. Good

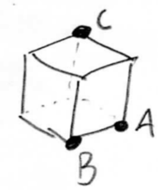


figure 1.3

c) Consider a regular dodecahedron, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why? This satisfies axioms 2 & 3, for similar reasons as above with the cube.

Good

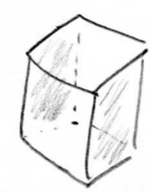
All three fail axiom 1, as ther are always examples where adjacent vertices lie on 2 distinct faces.

4. a) Consider a regular tetrahedron, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?

It satisfies the elliptical parallel postulate, as every face shares vertices with every other face, and thus there are no parallel lines.

Great

b) Consider a cube, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?



This satisfies the Euclidean parallel postulate, as opposite sides and ONLY opposite sides are parallel, and every point not on one side l has only one side k that is parallel to l.

Yes

c) Consider a regular dodecahedron, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?

This satisfies no parallel postulate.
If you take one face F, there are points not on F that lie on exactly one parallel face, and other points that lie on multiple parallel faces.
Thus, no parallel postulate is satisfied.

Right!

5. Provide good justifications in the blanks below for the corresponding statements:

Theorem: If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$

Statement	**Reason**
Let $\triangle ABC$ be a triangle such that $\overline{AB} \cong \overline{AC}$.	Given
We must prove that $\angle ABC \cong \angle ACB$.	Because that's what we must prove.
Let D be the point in the interior of $\angle BAC$ such that $\overrightarrow{AD}$ is the bisector of $\angle BAC$.	If D is A point, not THE point, true by Bisector Thm.
There is a point E at which the ray $\overrightarrow{AD}$ intersects the segment $\overline{BC}$.	Cross bar theorem.
Then $\triangle BAE \cong \triangle CAE$	SAS postulate
and so $\angle ABE \cong \angle ACE$.	Definition of congruency of triangles.
This completes the proof because $\angle ACE = \angle ACB$ and $\angle ABE = \angle ABC$. $\square$	Definition of angles & betweeness and collinearity of C, E, B