Problem Set 2 **Set Theory & Topology** **Due 1/29/20**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker 2.1.7] For $i = 1, 2, ..., n$ let $I_n = (a_i, b_i)$ be an open interval. Show that $\bigcap\{I_i : i = 1, 2, ..., n\}$ is either the empty set or an open interval.

2. [Baker 2.1.8] Use Definition 2.1.6 to show that $f(x) = \begin{cases} -3 & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$ is not a continuous function.

3. [Baker 2.1.9] Use Definition 2.1.6 to show that $f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is not a continuous function.

4. [Baker 2.1.10] Complete the proof of Theorem 2.1.8.

5. [Baker 2.2.10] Show that the collection $\mathscr{C}$ given in Example 2.2.3 is a topology for $\mathbb{R}$.

6. [Baker 2.2.12] Determine if the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 2 & \text{if } x > 1 \\ -2 & \text{if } x \leq 1 \end{cases}$$

   is

   (a) $\mathscr{U} - \mathscr{U}$ continuous
   (b) $\mathscr{U} - \mathscr{H}$ continuous
   (c) $\mathscr{U} - \mathscr{C}$ continuous
   (d) $\mathscr{H} - \mathscr{U}$ continuous
   (e) $\mathscr{H} - \mathscr{H}$ continuous
   (f) $\mathscr{C} - \mathscr{H}$ continuous
   (g) $\mathscr{C} - \mathscr{C}$ continuous

7. [Baker 2.3.14] Let A and B be subsets of a topolpogical space $(X, \mathscr{T})$. Show that $(X - \text{Cl}(A)) \cup (X - \text{Cl}(B)) \subseteq X - \text{Cl}(A \cap B)$. Find an example that shows these sets are not in general equal.

8. [Baker 2.3.15] Let A and B be subsets of a topological space $(X, \mathscr{T})$. Show that

$$X - \text{Cl}(A \cup B) = (X - \text{Cl}(A)) \cap (X - \text{Cl}(B)).$$