Problem Set 6 **Set Theory & Topology** **Due 4/8/20**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Five of these problems will be selected (by Jon) for grading, with each worth 4 points.

1. [Baker 6.1.7] Let $\mathbb{R}$ have the usual topology. Show (directly from the definition) that an open interval (a, b) is not compact.

2. [Baker 6.1.18] Give an example of a topology $\mathscr{T}$ on $\mathbb{R}$ for which there is a closed and bounded subset A of $\mathbb{R}$ that is not compact.

3. [Baker 6.1.18] Give an example of a topology $\mathscr{S}$ on $\mathbb{R}$ for which there is a compact subset A of $\mathbb{R}$ that is neither closed nor bounded.

4. [Baker 6.2.5] Let X be a compact topological space and let Y be a Hausdorff topological space. Show that if $f : X \to Y$ is a continuous function, then the inverse image of each compact subset of Y is a compact subset of X.

5. [Baker 6.2.8] Prove that compactness is a topological property.