Problem Set 8Set Theory & TopologyDue 4/20/20

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Five of these problems will be selected (by Jon) for grading, with each worth 4 points.

- 1. [Baker Th. 8.1.9] Let (X, d) be a metric space and let $U \subseteq X$. Then U is open with respect to the metric topology iff for each $x \in U$, there exists r > 0 such that $B_r(x) \subseteq U$.
- 2. [Baker 8.2.5] Complete the proof of Theorem 8.2.5.
- 3. [Baker 8.2.5] Let (X, d) and (Y, e) be metric spaces. Prove that a function $f : X \to Y$ is continuous iff, for each $x \in X$ and each $\varepsilon > 0$, there exists a $\delta > 0$ such that $f(B^d_{\delta}(x) \subseteq B^e_{\varepsilon}(f(x)))$.
- 4. [Baker 8.2.6] Complete the proof of Theorem 8.2.13.
- 5. [Baker Th. 8.3.12] Let *X* be a topological space with $A \subseteq X$ and $x \in X$. If there is a sequence in $A \{x\}$ which converges to *x*, then *x* is a limit point of *A*.