1.2 Parity

Definition: Call an integer m **even** iff it is equal to $2n$ for some integer n.
Definition: Call an integer m **odd** iff it is equal to $2n + 1$ for some integer n.

Exercises

1. If $n \in \mathbb{Z}$ is even, then n^2 is even.
2. If $n \in \mathbb{Z}$ is odd, then n^2 is odd.
3. If $n \in \mathbb{Z}$ is odd and $m \in \mathbb{Z}$ is even, then $n + m$ is odd.
4. If $n, m \in \mathbb{Z}$ are odd, then $n \cdot m$ is odd.
5. If $n^2 \in \mathbb{Z}$ is even, then n is even.
6. The cube of an even number is even.
7. The cube of an odd number is odd.
8. The product of any two consecutive integers is even.
9. The sum of any two consecutive integers is odd.
10. The sum of any two non-consecutive integers is even.

1.3 Beyond Parity

Definition: Call $m \in \mathbb{Z}$ **threven** iff $m = 3n$ for some $n \in \mathbb{Z}$.
Definition: Call $m \in \mathbb{Z}$ **throdd** iff $m = 3n + 1$ for some $n \in \mathbb{Z}$.
Definition: Call $m \in \mathbb{Z}$ **throddodd** iff $m = 3n + 2$ for some $n \in \mathbb{Z}$.

Exercises

1. The sum of two threven integers is threven.
2. The sum of two throdd integers is throddodd.
3. The sum of a throdd and a throddodd integer is threven.
4. The product of a threven integer with a throdd integer is threven.
5. The product of any three consecutive integers is threven.
6. The square of a threven integer is threven.
7. The square of a throdd integer is throdd.
8. The square of a throddodd integer is throdd.
9. There is no integer whose square is throddodd.
10. There is no integer which is both threven and throdd.