Problem Set 3   **Foundations**   **Due 2/12/21**

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submit your work as a pdf on the course Moodle page.

1. $\sqrt{5}$ is irrational.

2. The sum of two rational numbers is rational.

3. The sum of a rational and an irrational number is irrational.

4. The sum of two irrational numbers is irrational.

5. The product of a rational and an irrational is irrational.

6. The product of a non-zero rational and an irrational is irrational.

7. Consider the formula $1 + 2 + 3 + \ldots + n = \frac{n^2+n+1}{2}$.

    (a) Write the formula in sigma notation.

    (b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.

    (c) Explain why mathematical induction does not prove that this formula is true for all $n \in \mathbb{N}$.

8. For any $n \in \mathbb{N}$, with $n \geq 1$,

$$\sum_{i=1}^{n} c = nc$$

9. For any $n \in \mathbb{N}$, with $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

10. For any integer n, the number $n^2 + n + 17$ is prime.