Exam 1 **Advanced Geometry** **2/19/21**

Don't panic.

1. (a) Briefly describe an example of a model for a geometry that satisfies the elliptic parallel postulate.

A Sphere in which points are defined as on the surface of the sphere and lines are circles on the sphere with maximum circumference - "great circles" satisfies this, as every line intersects with any different line.

(b) Briefly describe an example of a model for a geometry that satisfies the Euclidean parallel postulate.

The cartisian plane satisfies this, as for any line l on this plane, and point P on this plane, the only line which p lies on which is parralell to l has the same slope as l, and unique lines are defined by point + slope.

(c) Briefly describe an example of a model for a geometry that satisfies the hyperbolic parallel postulate.

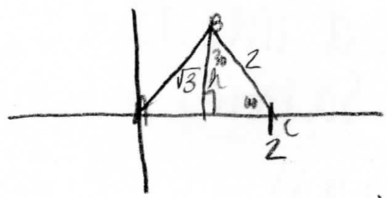
A klien disc, or bounded plane satisfies this postulate.

for any line l and point p, there are an infinite number of lines which p lies on that do not intersect l, as the slopes are simmilar enough that theoretical intersection would happen outside the bounded plane.

Excellent!

2. Explain why there's an issue with Euclid's Proposition 1, constructing an equilateral triangle given one of its sides, in the rational plane.

OK, well lets consider the equilateral triangle



the length of the triangle is the Rational #2

but we see that the height of the triangle is $\sqrt{3}$ which we proved in FOAM to be a irrational #. Since the height technically doesnt exist on the rational plane the triangle sadly falls apart

Excellent

3. Recall the three axioms of Incidence Geometry:

- **Incidence Axiom 1:** For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.

- **Incidence Axiom 2:** For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.

- **Incidence Axiom 3:** There exist three points that do not all lie on any one line.

Consider a geometry where points are ordered pairs of real numbers as in the Cartesian Plane, but lines are circles. Explain why this satisfies or fails to satisfy each of the three axioms for an incidence geometry.

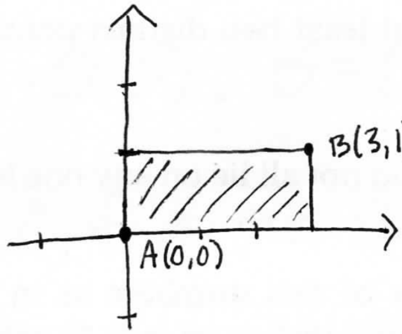
It fails IA1 because once you pick P and Q there are lots of circles through them:

It satisfies IA2 because every circle (presuming positive radius) has lots of points.

It satisfies IA3 because there are collections of points like (0,0), (1,0), and (2,0) that don't lie on any circle (because they're collinear in the Euclidean sense).

4. Let $A = (0, 0)$ and $B = (3, 1)$, and let ρ be the taxicab metric.

a) Describe all points on $\overline{AB}$.



all points

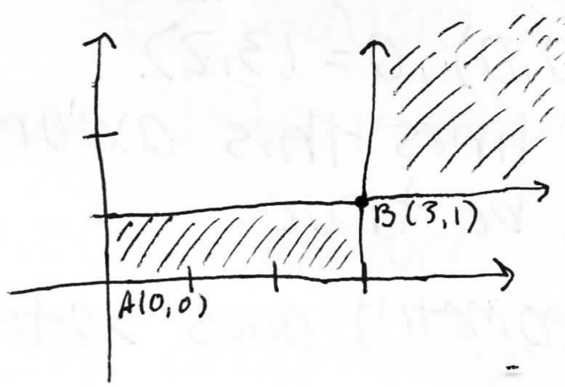
$0 \leq x \leq 3$

$0 \leq y \leq 1$

are between A and B.

Good

b) Describe all points on $\overrightarrow{AB}$.



5. Provide good justifications in the blanks below for the corresponding statements:

Theorem: If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$

Statement	**Reason**
Let $\triangle ABC$ be a triangle such that $\overline{AB} \cong \overline{AC}$.	Given
We must prove that $\angle ABC \cong \angle ACB$.	Because that's what we must prove.
Let D be the point in the interior of $\angle BAC$ such that $\overrightarrow{AD}$ is the bisector of $\angle BAC$.	Bisector Theorem, if D is a point not the point.
There is a point E at which the ray $\overrightarrow{AD}$ intersects the segment $\overline{BC}$.	True by the Crossbar Theorem
Then $\triangle BAE \cong \triangle CAE$	True by the SAS Postulate
and so $\angle ABE \cong \angle ACE$.	True by the definition of congruency of triangles
This completes the proof because $\angle ACE = \angle ACB$ and $\angle ABE = \angle ABC$. $\square$	True by the definition of angles and betweeness, and collinearity of C, E, B. $\square$

Good