Exam 1 **Advanced Geometry** **2/19/21**

Don't panic.

1. (a) Briefly describe an example of a model for a geometry that satisfies the elliptic parallel postulate.

   (b) Briefly describe an example of a model for a geometry that satisfies the Euclidean parallel postulate.

   (c) Briefly describe an example of a model for a geometry that satisfies the hyperbolic parallel postulate.

2. Explain why there's an issue with Euclid's Proposition 1, constructing an equilateral triangle given one of its sides, in the rational plane.

3. Recall the three axioms of Incidence Geometry:

   - **Incidence Axiom 1:** For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.

   - **Incidence Axiom 2:** For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.

   - **Incidence Axiom 3:** There exist three points that do not all lie on any one line.

   Consider a geometry where points are ordered pairs of real numbers as in the Cartesian Plane, but lines are circles. Explain why this satisfies or fails to satisfy each of the three axioms for an incidence geometry.

4. Let $A = (0, 0)$ and $B = (3, 1)$, and let ρ be the taxicab metric.

a) Describe all points on $\overline{AB}$.

b) Describe all points on $\overrightarrow{AB}$.

5. Provide good justifications in the blanks below for the corresponding statements:

Theorem: If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$

Statement	**Reason**
Let $\triangle ABC$ be a triangle such that $\overline{AB} \cong \overline{AC}$.	Given
We must prove that $\angle ABC \cong \angle ACB$.	Because that's what we must prove.
Let D be the point in the interior of $\angle BAC$ such that $\overrightarrow{AD}$ is the bisector of $\angle BAC$.	
There is a point E at which the ray $\overrightarrow{AD}$ intersects the segment $\overline{BC}$.	
Then $\triangle BAE \cong \triangle CAE$	
and so $\angle ABE \cong \angle ACE$.	
This completes the proof because $\angle ACE = \angle ACB$ and $\angle ABE = \angle ABC$. $\square$	