Examlet 2 Advanced Geometry 3/17/21

1. a) State the definition of a *scalene triangle*.

   b) State the definition of σ(△ABC).

   c) State the Saccheri-Legendre Theorem.

   d) State the Alternate Interior Angles Theorem

   e) State the definition of *defect*.

2. Which of the following are equivalent (given the other postulates of neutral geometry) to the Euclidean Parallel Postulate? Check all that apply.

☐ The double perpendicular construction

☐ The Saccheri-Legendre Theorem

☐ Existence of rectangles

☐ Euclid's Postulate V

☐ Converse of the Alternate Interior Angles Theorem

☐ If $\triangle ABC$ is a triangle, then $\sigma(\triangle ABC) = 180°$.

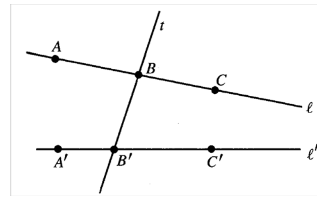
☐ Clairaut's Axiom

☐ Every triangle has defect 0˚.

☐ There exists a triangle whose defect is 0˚.

☐ The Universal Hyperbolic Theorem

3. Provide good justifications in the blanks below for the corresponding statements:



Proposition: If ℓ and ℓ' are two lines cut by a transversal t in such a way that a pair of alternate interior angles is congruent, then ℓ is parallel to ℓ'.

Statement:	Reason:
Let ℓ and ℓ' be two lines cut by transversal t such that a pair of alternate interior angles is congruent.	
Choose points A, B, C, and A', B', C' as in the figure above. Suppose $\angle A'B'B \cong \angle B'BC$.	
We must prove that ℓ is parallel to ℓ'. Suppose there exists a point D such that D lies on both ℓ and ℓ'.	
If D lies on the same side of t as C, then $\angle A'B'B$ is an exterior angle for $\triangle BB'D$,	
while $\angle B'BC$ is a remote interior angle for $\triangle BB'D$.	
This is a contradiction.	
In case D lies on the same side of t as A, then $\angle B'BC$ is an exterior angle and $\angle A'B'B$ is a remote interior angle for $\triangle BB'D$,	
and again we have a contradiction.	
Since D must lie on one of the two sides of t,	
we are forced to conclude that the proposition holds.	

4. Provide good justifications in the blanks below for the corresponding statements:

Proposition: If there exists one line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0, then for every line ℓ and for every external point P there exist at least two lines that pass through P and are parallel to ℓ.

Statement:	Reason:
S'pose there exists a line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0.	Hypothesis
Then the Euclidean Parallel Postulate fails.	
No rectangle exists.	
Let ℓ be a line and P an external point.	
We must prove that there are at least two lines through P that are both parallel to ℓ. Drop a perpendicular to ℓ through P and call the foot of that perpendicular Q.	
Let m be the line through P that is perpendicular to $\overleftrightarrow{PQ}$.	
Choose a point R on ℓ that is different from Q and let t be the line through R that is perpendicular to ℓ.	
Drop a perpendicular from P to t and call the foot of the perpendicular S.	
Now $\square PQRS$ is a Lambert quadrilateral.	
But $\square PQRS$ is not a rectangle (reason?), so $\angle QPS$ is not a right angle and $\overleftrightarrow{PS} \neq m$.	
Nevertheless $\overleftrightarrow{PS}$ is parallel to ℓ,	
so our proof is complete.	Because our proof is complete.

5. Explain, as if to someone intelligent but without any math background beyond high school, why if there exists one triangle whose defect is 0°, then every triangle has a defect of 0°.