Exam 3 Calculus 2 4/1/22

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Determine the exact sum of the geometric series

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$$

2. Find the first 3 partial sums of the series

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)!}$$

3. Find the 5^{th} degree MacLaurin polynomial for $f(x) = e^x$.

4. Determine whether the series $\sum_{n=2}^{\infty} \frac{3}{n \ln n}$ converges or diverges.

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{\sqrt{n}}$ converges or diverges.

6. Write the first 4 non-zero terms of the Taylor Series for $f(x) = \sin x$ centered at $x = \frac{\pi}{2}$.

7. Biff is a Calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this series stuff is crazy. So we were supposed to say if $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges or diverges, right? So I said it diverges by comparison to $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$, but this other person said it converges by geometric. So I figure that makes it a tie, right?"

   Help Biff out by explaining the validity of his two different justifications. You do not need to say whether the series converges or not; you're just commenting on the proposed justifications.

8. Use a Maclaurin polynomial with at least 4 terms to approximate $\sqrt[3]{e}$.

9. Does $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$ converge when $x = 1$?

10. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$.

Extra Credit [5 points possible]: Find the exact value of $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \cdots + \frac{(-1)^n}{(2n)!} + \cdots$