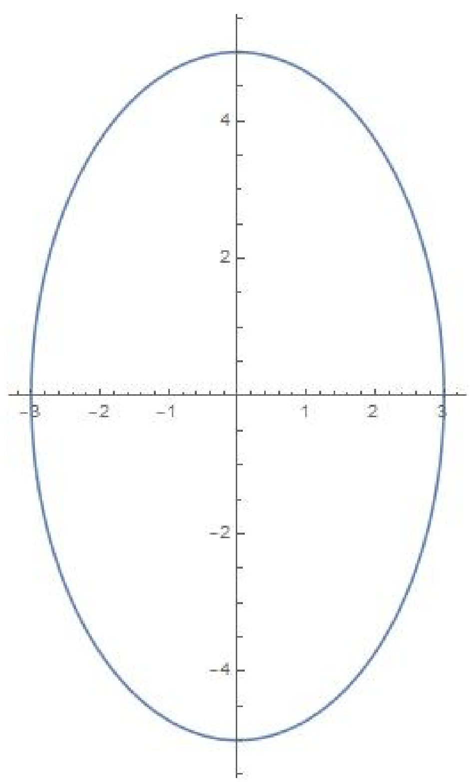
**Exam 4** **Calculus 2** **4/21/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Convert the point with rectangular coordinates $(-5, 5)$ to polar coordinates $(r, \theta)$.

2. Find an equation for the ellipse shown:



3. Consider the curve defined by the parametric equations $x(t) = t^3 - 5t$ and $y(t) = 8t^2$. Set up an integral for the length of the loop of this curve.

4. Set up an integral for the area of the region inside the curve with polar equation $r = 6\sin(5\theta)$.

5. Identify the graph of $y^2 - x^2 - 10y + 4x - 15 = 0$ as a parabola, hyperbola, or ellipse, give coordinates of its vertices, and sketch a decent graph.

6. Write an integral for the area of the region inside the inner loop of $r = 1 + 2\cos(\theta)$.

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. I am just totally confused. So like, I know with normal stuff, like, negative $x$ is like the left half, right? And negative $y$ is the bottom half, right? But I asked what half is negative with, like, this new $r$ thingy, right? And the professor just looked at me funny, in front of like 300 people in the lecture, right? So I pretty much died and he just went on. I think I better drop."

   Help Bunny out by explaining where points with negative $r$-values can be located.

8. Find the exact coordinates of all points on the graph of the curve with parametric equations $x(t) = t^3 - 6t, y(t) = t^2 - 5$ where the tangent line is vertical.

9. Find the exact $(x, y)$ coordinates of all point(s) with horizontal tangent lines on the cardioid with polar equation $r = 1 + \cos\theta$.

10. Find the area enclosed by the loop of the curve with parametric equations $x(t) = t^3 - 3t, y(t) = t^2 + t + 1$

Extra Credit [5 points possible]: [Rogawski/Adams] For $a > 0$, a lemniscate curve is the set of points such that the product of the distances from $P$ to $(a, 0)$ and $(-a, 0$ is $a^2$. Show that the equation of the lemniscate is

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$$