

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submit your work either on paper or as a pdf on the course Moodle page.

1. If  $p, q \in \mathbb{Z}$  are both even, then the product of  $p$  and  $q$  is even.
2. If  $p, q \in \mathbb{Z}$  with the product of  $p$  and  $q$  even, then  $p$  and  $q$  are even.
3. The cube of an odd integer is odd.
4. The sum of a throddodd and a throddodd integer is throdd.
5. If  $n \in \mathbb{Z}$  is throdd, then  $-n$  is throdd.
6. The product of a threven and a throdd integer is threven.
7. The sum of any three consecutive integers is threven
8. Critique the following proof of the proposition "The sum of any two throdd integers must be even":

Well, let  $m$  and  $n$  be throdd integers, so  $m = 3a + 1$  and  $n = 3a + 1$ , where  $a \in \mathbb{Z}$ . Then  $m + n = (3a + 1) + (3a + 1) = 6a + 2 = 2(3a + 1)$ , and  $3a + 1$  is an integer by closure, so  $m + n$  is 2 times an integer and thus even by definition.  $\square$