Problem Set 10 **Foundations** **Due 4/25/22**

Five of these problems will be graded, with each problem worth 4 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submit as a pdf via Moodle or on paper.

1. Do the Combinatorics assignment on WeBWorK, available via http://crlinwebwork2.coe.edu/webwork2/MTH-215/Combinatorics .

2. The Banana Theorem: Let A be a set with n elements of k different types (such that elements of the same type are regarded as indistinguishable from one another for purposes of orderings). Let n_i be the number of elements of type i for each integer i from 1 to k. Then the number of different arrangements of the elements in A will be
$$\frac{n!}{\prod_{i=1}^{k}(n_i!)}.$$

3. If a jar contains five balls, one red, three blue, and one white, and two balls are drawn at random from the jar (without replacement), what is the probability that one of the balls drawn is blue if you know that the other one is not blue?

4. If a jar contains four balls, one red, three blue, and one white, and two balls are drawn at random from the jar (without replacement), what is the probability that the red ball is drawn if you know that not both balls drawn were blue?

5. "In the jungle, you must wait, until the dice read 5 or 8" – from the movie Jumanji What is the probability of a 5 or 8 total when two standard dice are rolled?

6. $\forall x, y \in N, x + (y + 0) = (x + y) + 0$

7. $\forall x, y, z \in N, x + (y + z) = (x + y) + z \Rightarrow x + (y + z') = (x + y) + z'$

8. $\forall x, y, z \in N, x + (y + z) = (x + y) + z$

9. $\forall y \in N, 0 + y = y + 0$

10. $\forall x, y \in N, x + y = y + x \Rightarrow x' + y = y + x'$

11. $\forall x, y \in N, x + y = y + x$

12. Using the definition of $S(A)$ from section 5.6, write $S(\emptyset), S(S(\emptyset)), S(S(S(\emptyset)))$, and $S(S(S(S(\emptyset))))$ explicitly. How many elements are in each of these sets?

13. With the understanding that $0' = 1, 1' = 2, 2' = 3$, and $3' = 4$, where these are elements of a Peano system, show that $2 + 2 = 4$.