Problem Set 1 **Set Theory & Topology** **Due 1/19/22**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker 1.R.14] Let $f : X \to Y$ be a function and let A and B be subsets of Y. If $f^{-1}(A) = f^{-1}(B)$, then $A = B$.
2. Let $f : X \to Y$ be an onto function and let A and B be subsets of Y. If $f^{-1}(A) = f^{-1}(B)$, then $A = B$.
3. [Baker 1.R.15] If $f : X \to Y$ is a function, then $f(X) = Y$.
4. [Baker 1.R.16] If $f : X \to Y$ is onto, then $f(X) = Y$.
5. [Baker 1.R.17] Inverse images of sets are only defined for one-to-one functions.
6. [Baker 1.R.18] If $f : X \to Y$ is a function, then $f^{-1}(Y) = X$.
7. [Baker 1.R.19] If $f : X \to Y$ is a function and U and V are subsets of X, then $f(U \cap V) = f(U) \cap f(V)$.
8. [Baker 1.R.20] If $f : X \to Y$ is a function and U and V are subsets of X, then $f(U \cap V) \subseteq f(U) \cap f(V)$.
9. Is $\bigcap_{n\in\mathbb{Z}^+} \left(\frac{-1}{n}, \frac{n+1}{n}\right)$ open in the usual topology on $\mathbb{R}$? Why or why not?
10. Is $\bigcup_{n\in\mathbb{Z}}(n, n+1)$ open in the usual topology on $\mathbb{R}$? Why or why not?
11. Is $\bigcap_{n\in\mathbb{Z}^+} (-n, n)$ open in the usual topology on $\mathbb{R}$? Why or why not?
12. Is $\bigcup_{n\in\mathbb{Z}^+}(n, 2n)$ open in the usual topology on $\mathbb{R}$? Why or why not?