Problem Set 4 **Set Theory & Topology** **Due 3/7/22**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Ten of these problems will be selected (by Jon) for grading, with each worth 2 points.

1. [Baker 3.2.2]
2. [Baker Ch 3 R12] If a function $f : \mathbb{R} \to \mathbb{R}$ is $\mathscr{C} - \mathscr{U}$ continuous, then f is $\mathscr{U} - \mathscr{U}$ continuous.
3. [Baker Ch 3 R13] Any two discrete topological spaces are homeomorphic.
4. [Baker Ch 3 R14] Any one-to-one, onto function between two discrete topological spaces is a homeomorphism.
5. [Baker Ch 3 R17] If $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ are homeomorphic topological spaces, then any one-to-one function from X onto Y is a homeomorphism.
6. Prove Theorem 10 in §4.1: Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces. If A and B are closed subsets of X and Y, respectively, then $A \times B$ is a closed subset of $X \times Y$.
7. Prove Lemma 5.1.10: Let U be a nonempty subset of $\mathbb{R}$ that is bounded below and let n be the greatest lower bound for U. If I is any open interval containing m, then $I \cap U \neq \varnothing$.
8. [Baker 5.2.7] Complete the proof of Theorem 5.2.1.
9. Prove that topological spaces $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ are connected iff $X \times Y$ with the product topology is connected.
10. Prove Theorem 5.3.4.