

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker Exercise 7.1.13] Let  $X$  be a  $T_1$  space. Prove that if  $A$  is a finite subset of  $X$ , then  $A$  does not have a limit point.
2. [Baker Exercise 7.1.14] Let  $X$  and  $Y$  be topological spaces and let  $f : X \rightarrow Y$  be a function. The *graph* of  $f$  is the set  $G(f) = \{(x, y) \in X \times Y : y = f(x)\}$ . Prove that if  $f$  is continuous and  $Y$  is a  $T_2$ -space, then  $G(f)$  is a closed subset of the product space  $X \times Y$ .
3. [Baker Exercise 8.2.9] Let  $(X, d)$  be a metric space. Show that the function  $e : X \times X \rightarrow \mathbb{R}$  given by  $e(x, y) = \min\{1, d(x, y)\}$  is a metric for  $X$ .
4. [Baker Exercise 8.2.10] Show that the metric topology induced by the metric  $e$  given in Exercise 9 is the same as the metric topology induced by  $d$ .