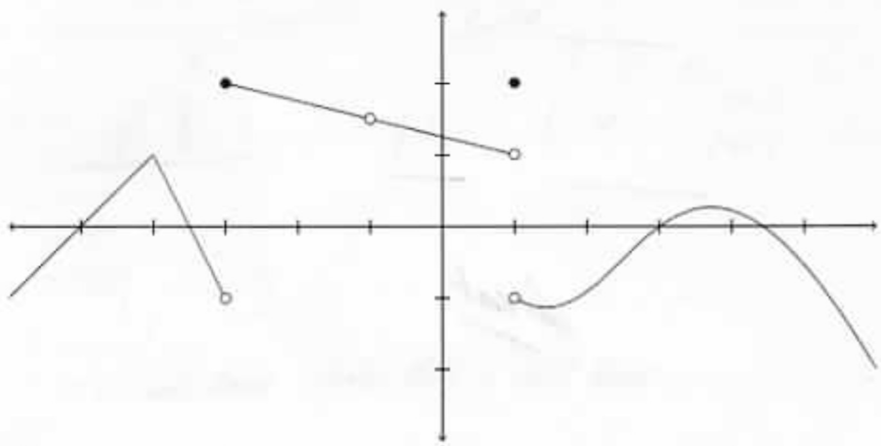
Exam 1 **Calc 1** **2/8/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



1. (a) What is $\lim_{x\to 1^+} f(x)$? $= -1$, as x approaches from the right Great

   (b) What is $\lim_{x\to 1^-} f(x)$? $= 1$, as x approaches from the left

   (c) What is $\lim_{x\to 1} f(x)$? = DNE, limits don't agree from the right and left

   (d) What is $\lim_{x\to -3^+} f(x)$? $= 2$, as x approaches from the right

   (e) What is $\lim_{x\to -3^-} f(x)$? $= -1$, as x approaches from the left

2. For which values of x does the function fail to be continuous?

The fuction fails to be continuous at: $x = -3$ because of jump discontinutity, $x = -1$ because there is no output value, and $x = 1$ because the $\lim_{x\to 1^-}$ and $\lim_{x\to 1^+}$ do not match the value of $f(z)$.

Excellent!

3. Evaluate $\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$.

$$\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$$

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x-1)}{\cancel{(x-5)}}$$

$$= \lim_{x \to 5} x - 1$$

$$= 5 - 1$$

$$= 4$$

Great!

4. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	5	4	6	1	3	2
$g(x)$	1	6	2	3	5	4

(a) $f(4)$ $= 1$

(b) $f(g(3))$ $= f(2) = 4$

Great!

(c) $g(6) - 1$ $= 4 - 1 = 3$

(d) $(f \circ g)(2)$ $= f(g(2)) = f(6) = 2$

(e) $(g \circ f)(2)$ $= g(f(2)) = g(4) = 3$

5. Evaluate $\lim_{x\to\infty} \frac{x^2-4}{2-7x-3x^2}$.

$$\lim_{x\to\infty} \frac{(x^2-4)\cdot\frac{1}{x^2}}{(2-7x-3x^2)\frac{1}{x^2}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{1-\frac{4}{x^2}}{\frac{2}{x^2}-\frac{7}{x}-3}$$

$$= -\frac{1}{3}$$

So $\lim_{x\to\infty} \frac{x^2-4}{2-7x-3x^2} = -\frac{1}{3}$

Great

6. A 2018 Tesla Model 3 Dual Motor Performance can accelerate for 0 to 60 miles per hour in 3.2 seconds. The distance from the car to a brick wall is given by $d(t) = 220 - 13.75t^2$ for values of t between 0 and 4. Find the average velocity of the Tesla over the time period **ending** when $t = 4$ and lasting

(a) 0.5 seconds

$$\frac{\overset{51.5625}{220-13.75(3.5)^2} - \overset{0}{220-13.75(4)^2}}{3.5-4} \rightarrow \boxed{-103.125 \text{ mph}}$$

(b) 0.1 seconds

Great

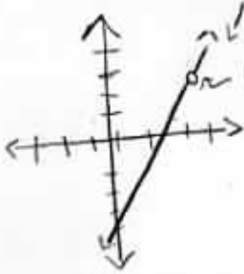
$$\frac{220-13.75(3.9)^2}{3.9-4} \rightarrow -\boxed{108.625}\text{ mph}$$

(c) 0.01 seconds

$$\frac{220-13.75(3.99)^2}{3.99-4} \rightarrow -109.8625 \text{ mph}$$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Calc is totally killing me. I thought it would be easy because of multiple choice, right? But it's like they're all trick questions. There was this one, like, for how many different inputs closer and closer to something do you have to get outputs closer and closer to something for you to know that's what the limit is, right? So I said 3 because that's how many they used in the online homework, so that's pretty simple, right? But they said it's none of the above, which is pretty much crap, because it's gotta be *something*, right?"

Help Biff by explaining as clearly as you can the answer to his question.



approaching there is ∞ number between 2.9 and 3

(3,3)

The reason is that it is none of the above is because even though for examples we use 3 numbers when talking about the limit we are talking about when x is approching said number and there is an infinite amount of numbers aprouching a number so you can use as many numbers as you would like you would like to find a limit.

Yes!

8. Let $f(x) = \dfrac{2-x}{(x-1)^2(x-3)}$.

(a) Evaluate $\lim_{x\to 3^+} f(x)$.

$$\lim_{x\to 3^+} \frac{2-x}{(x-1)^2(x-3)}$$

$$\Rightarrow \lim_{x\to 3^+} \left(\frac{2-x}{(x-1)^2} \times \frac{1}{x-3}\right)$$

$$\lim_{x\to 3^+} \left(\frac{2-x}{(x-1)^2}\right) \lim_{x\to 3^+} \left(\frac{1}{x-3}\right)$$

$$\Rightarrow -\frac{1}{4} \cdot +\infty \qquad = \underline{-\infty}$$

$$\therefore \lim_{x\to 3^+} f(x) = -\infty$$

(b) Evaluate $\lim_{x\to 3^-} f(x)$.

$$\lim_{x\to 3^-} \frac{2-x}{(x-1)^2(x-3)}$$

$$\Rightarrow \lim_{x\to 3^-} \left(\frac{2-x}{(x-1)^2} \times \frac{1}{x-3}\right)$$

$$\rightarrow \lim_{x\to 3^-} \left(\frac{2-x}{(x-1)^2}\right) \lim_{x\to 3^-} \left(\frac{1}{x-3}\right)$$

$$\rightarrow \left(-\frac{1}{4}\right)(-\infty)$$

$$= \underline{+\infty}$$

So $\lim_{x\to 3^-} f(x) = +\infty$

(c) Evaluate $\lim_{x\to 3} f(x)$.

$$\lim_{x\to 3} \frac{2-x}{(x-1)^2(x-3)}$$

Since $\lim_{x\to 3^+} f(x) = -\infty$

$\lim_{x\to 3^-} f(x) = +\infty$

So $\underline{\lim_{x\to 3} f(x) = \text{DNE}}$

Great!

9. (a) Evaluate $\lim_{x \to \infty} \frac{9e^x}{1-7e^x}$

$$\lim_{x \to \infty} \frac{9e^x \cdot \frac{1}{e^x}}{1-7e^x \cdot \frac{1}{e^x}}$$

$$\Rightarrow \lim_{x \to \infty} \frac{9}{\cancel{\frac{1}{e^x}} - 7}$$

$$\Rightarrow -\frac{9}{7}$$

(b) Find a value of b for which $\lim_{x \to \infty} \frac{9e^x}{1-be^x} = 2$

$$\lim_{x \to \infty} \frac{9e^x}{1-be^x} = 2$$

$$\lim_{x \to \infty} \frac{(9e^x) \cdot \frac{1}{e^x}}{(1-be^x) \cdot \frac{1}{e^x}}$$

$$\Rightarrow \lim_{x \to \infty} \frac{9}{\cancel{\frac{1}{e^x}} - b}$$

$$-\frac{9}{b} = 2$$

So $b = -\frac{9}{2}$

Excellent!

10. Let $f(x) = x^2 - 2x$.

(a) Find the slope of the secant line through $(1, f(1))$ and $(2, f(2))$.

$f(1) = (1)^2 - 2(1) = -1$

$f(2) = 4 - 4 = 0$

$$m = \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{0 - -1}{2 - 1} = \frac{1}{1} = 1$$

(b) Find the slope of the secant line through $(1, f(1))$ and $(1 + h, f(1 + h))$.

$f(1+h) = (1+h)^2 - 2(1+h) = 1 + 2h + h^2 - 2 - 2h = h^2 - 1$

$$m = \frac{f(1+h) - f(1)}{1 + h - 1} = \frac{h^2 - 1 - -1}{h} = \frac{h^2}{h} = h$$

(c) Find the slope of the tangent line through $(1, f(1))$.

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1} = \lim_{h \to 0} h = 0$$

Part b