## Exam 2 <br> Calc 1 3/3/23

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.
2. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 6 | 3 | 5 | 1 | 4 | 2 |
| $g(x)$ | 2 | 1 | 6 | 3 | 5 | 4 |
| $f^{\prime}(x)$ | 2 | 4 | 1 | 5 | 7 | 8 |
| $g^{\prime}(x)$ | 5 | 9 | 7 | 11 | 2 | 12 |

(a) If $h(x)=f(x) \cdot g(x)$, what is $h^{\prime}(2)$ and why?
(b) If $h(x)=\frac{f(x)}{g(x)}$, what is $h^{\prime}(5)$ and why?
(c) If $h(x)=f(g(x))$, what is $h^{\prime}(3)$ and why?
3. Find the derivatives of the following functions:
(a) $f(x)=x^{2} \sin x$
(b) $g(x)=\frac{\sin x}{x^{2}}$
(c) $h(x)=\sin \left(x^{2}\right)$
4. Show why the derivative of $\tan x$ is $\sec ^{2} x$.
5. Use the definition of the derivative to find the derivative of $f(x)=\frac{1}{x}$.
6. State and prove the Product Rule.
7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why does calculus have to be so confusing, like, they're literally trying to kill us? The professor definitely said that the derivative of 5 is zero on Monday, but then totally said that the derivative of 5 is 5 on Wednesday. Is it really different on different days?"

Help Bunny by explaining as clearly as you can what's going on.
8. Use a local linearization for the function $f(x)=x^{2 / 3}$ to approximate $(8.1)^{2 / 3}$.
9. Find an equation of the line tangent to $2 x y+x^{2}-y^{3}=4$ at the point $(3,-1)$.

10. A train leaves Boston heading west at noon travelling $16 \mathrm{mi} / \mathrm{hr}$. A second train is heading toward Boston from the north at $24 \mathrm{mi} / \mathrm{hr}$ and will arrive at 3pm. How fast is the distance between the trains changing at 2 pm ?

Extra Credit (5 points possible):
Is there a time when the distance between the trains from $\# 10$ is changing at a rate of $0 \mathrm{mi} / \mathrm{hr}$ ? When?

