Exam 4 **Calc 1** **4/14/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Given the following information about a function $f(x)$ which is continuous and differentiable, identify the x-values of all relative maximums, relative minimums, and inflection points of the function.

interval	$f'(x)$	$f''(x)$
$(-\infty, -2)$	-	$+$
$(-2, 3)$	$+$	$+$
$(3, 5)$	$+$	-
$(5, \infty)$	-	-

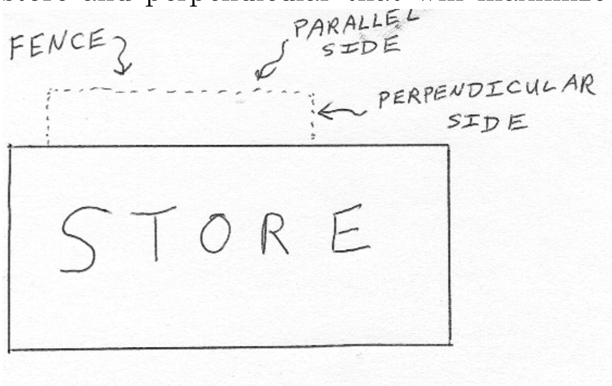
2. Find the interval(s) on which $f(x) = 3x^2 - 6x$ is decreasing.

3. Find the interval(s) where $g(x) = 14 + x^2 - x^3$ is concave up.

4. Find f if $f'(x) = 8x^3 + 14x + 10$ and $f(1) = -4$.

5. Let $f(x) = x^2 - 6x + 3$. Find the absolute maximum and absolute minimum values (heights) of f on $[1, 5]$.

6. [WW] The owner of a garden supply store wants to construct a fence to enclose a rectangular outdoor storage area adjacent to the store, using part of the side of the store (which is 270 feet long) for part of one of the sides. There are 450 feet of fencing available to complete the job. Find the length of the sides parallel to the store and perpendicular that will maximize the total area of the outdoor enclosure.



7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. I was understanding the derivative stuff pretty well, with the increasing and decreasing and stuff, right? But then the professor started talking about the second derivative, so like if increasing is increasing or decreasing, I guess? But I don't get it, and he was talking about, like, caves or something? Like cave up and cave down? But his accent is really tough, and I don't think I get it."

   Explain clearly to Biff what the second derivative tells us about the shape of a graph.

8. Find the x-values of *all* relative maximums and minimums of $y = \frac{x}{2} + \cos x$.

9. Use Newton's method to find the second and third approximation of a root of

$$x^3 - 4x - 3 = 0$$

starting with $x_1 = 2$ as the initial approximation.

10. [WW] A small resort is situated on an island that lies exactly 6 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 1.7 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?

Extra Credit (5 points possible): Find the point on the graph of $y = x^2 - 4x + 5$ closest to the origin.