Exam 1a Calculus 2 2/2/24

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Evaluate $\int \sin^4 \theta \cos \theta \, d\theta$

2. Evaluate $\int x e^x dx$

3. Write the appropriate form for a partial fractions decomposition of the function

$$\frac{2(x+1)}{(x-2)^2(x-1)^2(x^2+2)^2}$$

4. Evaluate
$$\int_{e}^{e^{5}} \frac{dx}{x\sqrt{\ln x}}$$

5. Evaluate
$$\int_3^\infty e^{p/2} dp$$

6. Evaluate
$$\int \frac{x^3}{\sqrt{1-x^2}} dx$$

7. Star is a calculus student at Enormous State University, and they're having some trouble. Star says "Yikes! Calc 2 is like a totally different thing than Calc 1. These problems are so long! And sometimes I wonder why they pick the things they do, like for a trig sub one, why do they do $x = \sin \theta$? Would it work if you did $x = \cos \theta$?"

Help Star out. Explain to them as clearly as possible whether their alternative works well, and why.

8. Derive Line 87 from the Table of Integrals,

$$\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1 - u^2} + C$$

9. Derive Line 30 from the Table of Integrals,

$$\int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

10. Evaluate
$$\int \frac{1}{1+x^3} dx$$
 [Hint: $1+x^3 = (1+x)(1-x+x^2)$]

Extra Credit [5 points possible]: Evaluate $\int \frac{x^2}{\sqrt{1-x}} dx$